

Déterminez une fonction  $F$  qui soit une primitive de la fonction  $f$ ,

Trois propriétés très souvent utilisées sont : (c.f. cours page 23)

I)  $\int \lambda \cdot g(f(x)) \cdot f'(x) dx = \lambda \cdot G(f(x)) + C$  où  $G$  est une primitive de  $g$ .

II)  $\int \lambda \cdot f^n(x) \cdot f'(x) dx = \lambda \cdot \frac{f^{n+1}(x)}{n+1} + C$  où  $n \in \mathbb{R} \setminus \{-1\}$ .

III)  $\int \lambda \cdot (f(x))^{-1} \cdot f'(x) dx = \lambda \cdot \ln|f(x)| + C$

De plus on sait que :  $\int \lambda \cdot x^n dx = \lambda \cdot \frac{x^{n+1}}{n+1} + C$  si  $n \neq -1$  et  $\int \lambda \cdot x^{-1} dx = \lambda \cdot \ln(|x|) + C$   $C \in \mathbb{R}$

1) $f(x) = x - 3 \cdot x^{-2}$	$F(x) = \frac{x^2}{2} - 3 \cdot \frac{x^{-1}}{-1} + C = \frac{x^2}{2} + \frac{3}{x} + C$
2) $f(x) = 2x + 1 - x^{-2}$	$F(x) = x^2 + x - \frac{x^{-1}}{-1} + C = x^2 + x + \frac{1}{x} + C$
3) $f(x) = \frac{1}{\lambda} \cdot \frac{(3x+2)^6}{f^n} \cdot \frac{(3x+2)'}{f'}$	$F(x) = \frac{1}{21} \cdot (3x+2)^7 + C$
4) $f(x) = \frac{(-1)}{\lambda} \cdot \frac{\cos^4(x)}{f^n} \cdot \frac{(\cos(x))'}{f'}$	$F(x) = -\frac{1}{5} \cdot \cos^5(x) + C$
5) $f(x) = \frac{(x^2+x+3)^{-2}}{f^n} \cdot \frac{(x^2+x+3)'}{f'}$	$F(x) = \frac{(x^2+x+3)^{-1}}{-1} + C = -\frac{1}{x^2+x+3} + C$
6) $f(x) = \frac{(1/2)}{\lambda} \cdot \frac{(x^2-2x+4)^{-1}}{f^{-1}} \cdot \frac{(x^2-2x+4)'}{f'}$	$F(x) = \ln( x^2-2x+4 )/2 + C$
7) $f(x) = \frac{(x^2-x-2)^{-1}}{f^{-1}} \cdot \frac{(x^2-x-2)'}{f'}$	$F(x) = \ln( x^2-x-2 ) + C$
8) $f(x) = \frac{(1-\cos(x))^1}{f^n} \cdot \frac{(1-\cos(x))'}{f'}$	$F(x) = (1-\cos(x))^2/2 + C$
9) $f(x) = \frac{(1/4)}{\lambda} \cdot \frac{(4 \cdot \sin(x)-1)^{-3}}{f^n} \cdot \frac{(4 \cdot \sin(x)-1)'}{f'}$	$F(x) = \frac{-1}{8 \cdot (4 \cdot \sin(x)-1)^2} + C$
10) $f(x) = 1 + \tan^2(2x)$	$F(x) = \tan(2x)/2 + C$ c.f. ex. 18 série 5
11) $f(x) = \frac{(1/2)}{\lambda} \cdot \frac{(2x+1)^3}{f^n} \cdot \frac{(2x+1)'}{f'}$	$F(x) = \frac{1}{8} \cdot (2x+1)^4 + C$
12) $f(x) = \frac{(1/2)}{\lambda} \cdot \frac{(x^2-4)^{-1}}{f^{-1}} \cdot \frac{(x^2-4)'}{f'}$	$F(x) = \ln( x^2-4 )/2 + C$
13) $f(x) = \frac{(\ln( x ))^{-1}}{f^{-1}} \cdot \frac{1}{f' \cdot x}$	$F(x) = \ln( \ln( x ) ) + C$

14) $f(x) = 2 \cdot \underset{\lambda}{(x^2 + x + 1)^{-1}} \cdot \underset{f^{-1}}{(x^2 + x + 1)'} \cdot \underset{f'}{(x^2 + x + 1)}$	$F(x) = 2 \cdot \ln( x^2 + x + 1 ) + C$
15) $f(x) = 2 \cdot x^{-3} + \frac{x^{-2}}{2}$	$F(x) = 2 \cdot \frac{x^{-2}}{-2} + \frac{1}{2} \cdot \frac{x^{-1}}{-1} + C = -\frac{1}{x^2} - \frac{1}{2x} + C$
16) $f(x) = \ln( x ) \cdot \ln( x )$	$F(x) = \ln^2( x )/2 + C$
17) $f(x) = \exp(\sqrt{2x}) \cdot (\sqrt{2x})'$	$F(x) = \exp(\sqrt{2x}) + C = e^{\sqrt{2x}} + C$
18) $f(x) = (\sin(x))^2 \cdot (\sin(x))'$	$F(x) = \frac{2}{3} \cdot (\sin(x))^3 + C = \frac{2}{3} \cdot \sqrt{(\sin(x))^3} + C$
19) $f(x) = (-1) \cdot \exp(1/x) \cdot (1/x)'$	$F(x) = -\exp(1/x) + C = -e^{1/x} + C$
20) $f(x) = x^{\frac{1}{3}}$	$F(x) = x^{\frac{2}{3}} / (2/3) + C = (3/2) \cdot \sqrt[3]{x^2} + C$
21) $f(x) = \frac{x^2 + 2x + 1}{x} = x + 2 + \frac{1}{x}$	$F(x) = \frac{x^2}{2} + 2x + \ln( x ) + C$
22) $f(x) = (1/3) \cdot \exp(x^3) \cdot (x^3)'$	$F(x) = (1/3) \cdot \exp(x^3) + C = (1/3) \cdot e^{x^3} + C$
23) $f(x) = x^{\frac{1}{3}} + x^{\frac{1}{3}}$	$F(x) = \frac{3}{4} \cdot x^{\frac{4}{3}} + \frac{3}{2} \cdot x^{\frac{2}{3}} + C = \frac{3}{4} \cdot \sqrt[3]{x^4} + \frac{3}{2} \cdot \sqrt[3]{x^2} + C$
24) $f(x) = 2 \cdot x^{\frac{1}{2}} + \sqrt{2} \cdot x^{\frac{1}{2}}$	$F(x) = \frac{4}{3} x^{\frac{3}{2}} + \frac{2 \cdot \sqrt{2}}{3} \cdot x^{\frac{3}{2}} + C = \left( \frac{4}{3} + \frac{2 \cdot \sqrt{2}}{3} \right) \cdot \sqrt{x^3} + C$
25) $f(x) = \frac{1}{2a} \cdot (ax^2 + b)^{\frac{1}{3}} \cdot (ax^2 + b)'$ , $a \neq 0$	$F(x) = \frac{1}{2a} \cdot \frac{3}{4} (ax^2 + b)^{\frac{4}{3}} + C = \frac{3}{8a} \cdot \sqrt[3]{(ax^2 + b)^4} + C$
26) $f(x) = (x^2 + x + 1)^{-\frac{1}{2}} \cdot (x^2 + x + 1)'$	$F(x) = 2 \cdot (x^2 + x + 1)^{\frac{1}{2}} + C = 2 \cdot \sqrt{x^2 + x + 1} + C$
27) $f(x) = (9 + x^3)^{-\frac{1}{2}} \cdot (9 + x^3)'$	$F(x) = 2 \cdot (9 + x^3)^{\frac{1}{2}} + C = 2 \cdot \sqrt{9 + x^3} + C$
28) $f(x) = (1/5) \cdot (5x^3 + 8)^{-\frac{1}{2}} \cdot (5x^3 + 8)'$	$F(x) = (1/5) \cdot 2 \cdot (5x^3 + 8)^{\frac{1}{2}} + C = (2/5) \cdot \sqrt{5x^3 + 8} + C$
29) $f(x) = (x^3 + x + 2)^{\frac{1}{2}} \cdot (x^3 + x + 2)'$	$F(x) = \frac{2}{3} \cdot \sqrt{(x^3 + x + 2)^3} = \frac{2}{3} \cdot (x^3 + x + 2) \cdot \sqrt{x^3 + x + 2} + C$
30) $f(x) = (x + 2\sqrt{x})^2 = x^2 + 4x^{\frac{3}{2}} + 4x$	$F(x) = \frac{1}{3} x^3 + 4 \cdot \frac{2}{5} \cdot x^{\frac{5}{2}} + 2x^2 + C = \frac{1}{3} x^3 + \frac{8}{5} \sqrt{x^5} + 2x^2 + C$