

Déterminez une fonction F qui soit une primitive de la fonction f ,

Trois propriétés très souvent utilisées sont : (c.f. cours page 23)

I) $\int \lambda \cdot g(f(x)) \cdot f'(x) dx = \lambda \cdot G(f(x)) + C$ où G est une primitive de g .

II) $\int \lambda \cdot f^n(x) \cdot f'(x) dx = \lambda \cdot \frac{f^{n+1}(x)}{n+1} + C$ où $n \in \mathbb{R} \setminus \{-1\}$.

III) $\int \lambda \cdot (f(x))^{-1} \cdot f'(x) dx = \lambda \cdot \ln|f(x)| + C$

De plus on sait que : $\int \lambda \cdot x^n dx = \lambda \cdot \frac{x^{n+1}}{n+1} + C$ si $n \neq -1$ et $\int \lambda \cdot x^{-1} dx = \lambda \cdot \ln(|x|) + C$ $C \in \mathbb{R}$

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| 1) $f(x) = x - 3 \cdot x^{-2}$ | $F(x) = \frac{x^2}{2} - 3 \cdot \frac{x^{-1}}{-1} + C = \frac{x^2}{2} + \frac{3}{x} + C$ |
| 2) $f(x) = 2x + 1 - x^{-2}$ | $F(x) = x^2 + x - \frac{x^{-1}}{-1} + C = x^2 + x + \frac{1}{x} + C$ |
| 3) $f(x) = \frac{1}{\lambda} \cdot \frac{(3x+2)^6}{f^n} \cdot \frac{(3x+2)'}{f'}$ | $F(x) = \frac{1}{21} \cdot (3x+2)^7 + C$ |
| 4) $f(x) = \frac{(-1)}{\lambda} \cdot \frac{\cos^4(x)}{f^n} \cdot \frac{(\cos(x))'}{f'}$ | $F(x) = -\frac{1}{5} \cdot \cos^5(x) + C$ |
| 5) $f(x) = \frac{(x^2+x+3)^{-2}}{f^n} \cdot \frac{(x^2+x+3)'}{f'}$ | $F(x) = \frac{(x^2+x+3)^{-1}}{-1} + C = -\frac{1}{x^2+x+3} + C$ |
| 6) $f(x) = \frac{(1/2)}{\lambda} \cdot \frac{(x^2-2x+4)^{-1}}{f^{-1}} \cdot \frac{(x^2-2x+4)'}{f'}$ | $F(x) = \ln(x^2-2x+4)/2 + C$ |
| 7) $f(x) = \frac{(x^2-x-2)^{-1}}{f^{-1}} \cdot \frac{(x^2-x-2)'}{f'}$ | $F(x) = \ln(x^2-x-2) + C$ |
| 8) $f(x) = \frac{(1-\cos(x))^1}{f^n} \cdot \frac{(1-\cos(x))'}{f'}$ | $F(x) = \frac{(1-\cos(x))^2}{2} + C$ |
| 9) $f(x) = \frac{(1/4)}{\lambda} \cdot \frac{(4 \cdot \sin(x)-1)^{-3}}{f^n} \cdot \frac{(4 \cdot \sin(x)-1)'}{f'}$ | $F(x) = \frac{-1}{8 \cdot (4 \cdot \sin(x)-1)^2} + C$ |
| 10) $f(x) = 1 + \tan^2(2x)$ | $F(x) = \tan(2x)/2 + C$ c.f. ex. 18 série 5 |
| 11) $f(x) = \frac{(1/2)}{\lambda} \cdot \frac{(2x+1)^3}{f^n} \cdot \frac{(2x+1)'}{f'}$ | $F(x) = \frac{1}{8} \cdot (2x+1)^4 + C$ |
| 12) $f(x) = \frac{(1/2)}{\lambda} \cdot \frac{(x^2-4)^{-1}}{f^{-1}} \cdot \frac{(x^2-4)'}{f'}$ | $F(x) = \ln(x^2-4)/2 + C$ |
| 13) $f(x) = \frac{(\ln(x))^{-1}}{f^{-1}} \cdot \frac{1}{f' \cdot x}$ | $F(x) = \ln(\ln(x)) + C$ |

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| 14) $f(x) = 2 \cdot \underset{\lambda}{(x^2+x+1)^{-1}} \cdot \underset{f'}{(x^2+x+1)'} \quad F(x) = 2 \cdot \ln(x^2+x+1) + C$ |
| 15) $f(x) = 2 \cdot x^{-3} + \frac{x^{-2}}{2} \quad F(x) = 2 \cdot \frac{x^{-2}}{-2} + \frac{1}{2} \cdot \frac{x^{-1}}{-1} + C = -\frac{1}{x^2} - \frac{1}{2x} + C$ |
| 16) $f(x) = \ln(x) \cdot \ln(x) \quad F(x) = \ln^2(x)/2 + C$ |
| 17) $f(x) = \exp(\sqrt{2x}) \cdot (\sqrt{2x})' \quad F(x) = \exp(\sqrt{2x}) + C = e^{\sqrt{2x}} + C$ |
| 18) $f(x) = (\sin(x))^2 \cdot (\sin(x))' \quad F(x) = \frac{2}{3} \cdot (\sin(x))^3 + C = \frac{2}{3} \cdot \sqrt{(\sin(x))^3} + C$ |
| 19) $f(x) = (-1) \cdot \exp(1/x) \cdot (1/x)' \quad F(x) = -\exp(1/x) + C = -e^{1/x} + C$ |
| 20) $f(x) = x^{\frac{1}{3}} \quad F(x) = x^{\frac{2}{3}} / (2/3) + C = (3/2) \cdot \sqrt[3]{x^2} + C$ |
| 21) $f(x) = \frac{x^2+2x+1}{x} = x+2+\frac{1}{x} \quad F(x) = \frac{x^2}{2} + 2x + \ln(x) + C$ |
| 22) $f(x) = (1/3) \cdot \exp(x^3) \cdot (x^3)' \quad F(x) = (1/3) \cdot \exp(x^3) + C = (1/3) \cdot e^{x^3} + C$ |
| 23) $f(x) = x^{\frac{4}{3}} + x^{\frac{2}{3}} \quad F(x) = \frac{3}{4} \cdot x^{\frac{4}{3}} + \frac{3}{2} \cdot x^{\frac{2}{3}} + C = \frac{3}{4} \cdot \sqrt[3]{x^4} + \frac{3}{2} \cdot \sqrt[3]{x^2} + C$ |
| 24) $f(x) = 2 \cdot x^{\frac{1}{2}} + \sqrt{2} \cdot x^{\frac{1}{2}} \quad F(x) = \frac{4}{3} x^{\frac{3}{2}} + \frac{2 \cdot \sqrt{2}}{3} \cdot x^{\frac{3}{2}} + C = \left(\frac{4}{3} + \frac{2 \cdot \sqrt{2}}{3} \right) \cdot \sqrt{x^3} + C$ |
| 25) $f(x) = \frac{1}{2a} \cdot (ax^2+b)^{\frac{1}{3}} \cdot (ax^2+b)', \quad a \neq 0 \quad F(x) = \frac{1}{2a} \cdot \frac{3}{4} (ax^2+b)^{\frac{4}{3}} + C = \frac{3}{8a} \cdot \sqrt[3]{(ax^2+b)^4} + C$ |
| 26) $f(x) = (x^2+x+1)^{\frac{1}{2}} \cdot (x^2+x+1)' \quad F(x) = 2 \cdot (x^2+x+1)^{\frac{1}{2}} + C = 2 \cdot \sqrt{x^2+x+1} + C$ |
| 27) $f(x) = (9+x^3)^{\frac{1}{2}} \cdot (9+x^3)' \quad F(x) = 2 \cdot (9+x^3)^{\frac{1}{2}} + C = 2 \cdot \sqrt{9+x^3} + C$ |
| 28) $f(x) = (1/5) \cdot (5x^3+8)^{\frac{1}{2}} \cdot (5x^3+8)' \quad F(x) = (1/5) \cdot 2 \cdot (5x^3+8)^{\frac{1}{2}} + C = (2/5) \cdot \sqrt{5x^3+8} + C$ |
| 29) $f(x) = (x^3+x+2)^{\frac{1}{3}} \cdot (x^3+x+2)' \quad F(x) = \frac{2}{3} \cdot \sqrt{(x^3+x+2)^3} = \frac{2}{3} \cdot (x^3+x+2) \cdot \sqrt{x^3+x+2} + C$ |
| 30) $f(x) = (x+2\sqrt{x})^2 = x^2 + 4x^{\frac{3}{2}} + 4x \quad F(x) = \frac{1}{3} x^3 + 4 \cdot \frac{2}{5} \cdot x^{\frac{5}{2}} + 2x^2 + C = \frac{1}{3} x^3 + \frac{8}{5} \sqrt{x^5} + 2x^2 + C$ |