Autre manière de calculer des primitives de fonctions, où l'on cherche a posteriori le nombre α pour que $F'(x) = f(x)$.

Déterminez une fonction F qui soit une primitive de la fonction f ,
c'est-à-dire **par définition** une fonction F telle que $F' = f$.
Quatre propriétés très souvent utilisées sont : (λ *est une constante quelconque.*)

I) $\int \lambda\cdot g(f(x))\cdot f'(x)dx = \alpha\cdot G(f(x)) + C$ où G est une primitive de g.

II) $\int \lambda\cdot f^n(x)\cdot f'(x)dx = \alpha\cdot f^{n+1}(x) + C$ où $n \in \mathbb{N}\setminus\{-1\}$.

III) $\int \lambda\cdot f^{-1}(x)\cdot f'(x)dx = \alpha\cdot\ln(|(f(x))|) + C$

IV) $\int \lambda\cdot e^{f(x)}\cdot f'(x)dx = \alpha\cdot e^{f(x)} + C$ Dans chaque cas, **α est une constante à déterminer**.

1)	$f(x) = \frac{5}{3}\cdot x^4 - \frac{3}{4}\cdot x^2 + 1$; $F(x) = \frac{1}{3}\cdot x^5 - \frac{1}{4}\cdot x^3 + x + C$ car $F'(x) = f(x)$
2)	$f(x) = (x+1)^2$; $F(x) = \alpha\cdot(x+1)^3$; $F'(x) = \alpha\cdot 3\cdot(x+1)^2 = f(x)$, donc $\alpha = \frac{1}{3}$ $F(x) = \frac{1}{3}\cdot(x+1)^3 + C$
3)	$f(x) = (2-x)^{12}$; $F(x) = \alpha\cdot(2-x)^{13}$; $F'(x) = \alpha\cdot 13\cdot(2-x)^{13}\cdot(-1) = f(x)$, donc $\alpha\cdot 13\cdot(-1) = 1$; $\alpha = -\frac{1}{13}$; $F(x) = -\frac{1}{13}\cdot(2-x)^{13} + C$
4)	$f(x) = (1-2x)^2$; $F(x) = \alpha\cdot(1-2x)^3$; $F'(x) = \alpha\cdot 3\cdot(1-2x)^2\cdot(-2) = f(x)$, donc $\alpha\cdot 3\cdot(-2) = 1$; $\alpha = -\frac{1}{6}$; $F(x) = -\frac{1}{6}\cdot(1-2x)^3 + C$
5)	$f(x) = (x^2-3x+1)^5\cdot(2x-3)$; $F(x) = \alpha\cdot(x^2-3x+1)^6$; $F'(x) = \alpha\cdot 6\cdot(x^2-3x+1)^5\cdot(2x-3) = f(x)$, donc $\alpha\cdot 6 = 1$; $\alpha = \frac{1}{6}$; $F(x) = \frac{1}{6}\cdot(x^2-3x+1)^6 + C$
6)	$f(x) = (x-1)^{-2}$; $F(x) = \alpha\cdot(x-1)^{-1}$; $F'(x) = \alpha\cdot(-1)\cdot(x-1)^{-1} = f(x)$, donc $\alpha = -1$; $F(x) = -(x-1)^{-1} + C = \frac{-1}{x-1} + C = \frac{1}{1-x} + C$
7)	$f(x) = (3x^2+1)^2\cdot 6x$; $F(x) = \alpha\cdot(3x^2+1)^3$; $F'(x) = \alpha\cdot 3\cdot(3x^2+1)^2\cdot 6x = f(x)$, donc $3\cdot\alpha = 1$; $\alpha = \frac{1}{3}$; $F(x) = \frac{1}{3}\cdot(3x^2+1)^3 + C$
8)	$f(x) = (1-x^2)^3\cdot 6x$; $F(x) = \alpha\cdot(1-x^2)^4$; $F'(x) = \alpha\cdot 4\cdot(1-x^2)^4\cdot(-2x) = f(x)$, donc $\alpha\cdot 4\cdot(-2) = 6$; $\alpha = -\frac{6}{8} = -\frac{3}{4}$; $F(x) = -\frac{3}{4}\cdot(1-x^2)^4 + C$
9)	$f(x) = -\frac{4}{x^4} - \frac{1}{x^3} + \frac{3}{x^5}$; $F(x) = \frac{4}{3x^3} + \frac{1}{2x^2} - \frac{3}{4x^4} + C$ car $F'(x) = f(x)$
10)	$f(x) = (1+2x^3)^{-2}\cdot 3x^2$; $F(x) = \alpha\cdot(1+2x^3)^{-1}$; $F'(x) = \alpha\cdot(-1)\cdot(1+2x^3)^{-2}\cdot 6x^2 = f(x)$, donc $\alpha\cdot(-1)\cdot 6 = 3$; $\alpha = -\frac{1}{2}$; $F(x) = -\frac{1}{2}\cdot(1+2x^3)^{-1} + C = \frac{-1}{2\cdot(1+2x^3)} + C$

11) $f(x) = (4x^2-5x)^2\cdot(16x-10)$; $F(x) = \alpha\cdot(4x^2-5x)^3$; $F'(x) = \alpha\cdot 3\cdot(4x^2-5x)^2\cdot(8x-5)$

donc $\alpha\cdot 3\cdot(8x-5) = 2\cdot(8x-5)$; $\alpha = \frac{2}{3}$; $F(x) = \frac{2}{3}\cdot(4x^2-5x)^3+C$

12) $f(x) = x^{\frac{3}{2}}$; $F(x) = \frac{2}{5}\cdot x^{\frac{5}{2}}+C = \frac{2}{5}\cdot\sqrt{x^5}+C$

13) $f(x) = \sqrt{x^2+1}\cdot x = (x^2+1)^{\frac{1}{2}}\cdot x$; $F(x) = \alpha\cdot(x^2+1)^{\frac{3}{2}}$; $F'(x) = \alpha\cdot\frac{3}{2}\cdot(x^2+1)^{\frac{1}{2}}\cdot 2x$

donc $\alpha = \frac{1}{3}$; $F(x) = \frac{1}{3}\cdot(x^2+1)^{\frac{3}{2}}+C = \frac{1}{3}\cdot\sqrt{(x^2+1)^3}+C$

14) $f(x) = \sqrt{x}-1/\sqrt{x}$; $F(x) = (2/3)\cdot\sqrt{x^3}-2\cdot\sqrt{x}+C$

15) $f(x) = \sin(3x)$; $F(x) = \alpha\cdot\cos(3x)$; $F'(x) = \alpha\cdot(-\sin(3x))\cdot 3$,

donc $\alpha = -\frac{1}{3}$; $F(x) = -\frac{1}{3}\cdot\cos(3x)+C$

16) $f(x) = 2\cdot\sin(x)+3\cdot\cos(x)$; $F(x) = -2\cdot\cos(x)+3\cdot\sin(x)+C$

17) $f(x) = \cos(x)-\sin^2(x)\cdot\cos(x)$; $F(x) = \sin(x)-\alpha\cdot\sin^3(x)$;

$F'(x) = \cos(x)-\alpha\cdot 3\cdot\sin(x)^2\cdot\cos(x)$, donc $\alpha = \frac{1}{3}$; $F(x) = \sin(x)-\frac{1}{3}\cdot\sin^3(x)+C$

18) $f(x) = \tan^2(x)$; $F(x) = \tan(x)-x+C$, c.f. table CRM ou le cours.

19) $f(x) = \frac{1}{2}\cdot\cos(4x)$; $F(x) = \alpha\cdot\sin(4x)$; $F'(x) = \alpha\cdot\cos(4x)\cdot 4$,

donc $\alpha\cdot 4 = \frac{1}{2}$; $\alpha = \frac{1}{8}$; $F(x) = \frac{1}{8}\cdot\sin(4x)+C$

20) $f(x) = \sin^5(x)\cdot\cos(x)$; $F(x) = \alpha\cdot\sin^6(x)$; $F'(x) = \alpha\cdot 6\cdot\sin^5(x)\cdot\cos(x)$,

donc $\alpha = \frac{1}{6}$; $F(x) = \frac{1}{6}\cdot\sin^6(x)+C$

21) $f(x) = \cos^2(x/2)\cdot\sin(x/2)$; $F(x) = \alpha\cdot\cos^3(x/2)$; $F'(x) = \alpha\cdot 3\cdot\cos^2(x/2)\cdot(-\sin(x/2))\cdot(1/2)$

donc $\alpha\cdot 3\cdot(-1)\cdot(1/2) = 1$; $\alpha = -\frac{2}{3}$; $F(x) = -\frac{2}{3}\cdot\cos^3(x/2)+C$

22) $f(x) = (1+\cos(x))^{-2}\cdot\sin(x)$; $F(x) = \alpha\cdot(1+\cos(x))^{-1}$; $F'(x) = \alpha\cdot(-1)\cdot(1+\cos(x))^{-2}\cdot(-\sin(x))$

donc $\alpha = 1$; $F(x) = (1+\cos(x))^{-1} = 1/(1+\cos(x))$

23) $f(x) = (2x-5)^{-1}$; $F(x) = \alpha\cdot\ln(|(2x-5)|)$; $F'(x) = \alpha\cdot\frac{1}{2x-5}\cdot 2$;

donc $\alpha = \frac{1}{2}$; $F(x) = \frac{1}{2}\cdot\ln(|(2x-5)|)+C$

24) $f(x) = \cos^{-1}(x)\cdot(-\sin(x))$; $F(x) = \ln(|(\cos(x))|)+C$

25) $f(x) = (x^2+1)^{-1}\cdot 3x$; $F(x) = \alpha\cdot\ln(|(x^2+1)|)$; $F'(x) = \alpha\cdot\frac{1}{x^2+1}\cdot 2x$;

donc $\alpha\cdot 2 = 3$; $\alpha = \frac{3}{2}$; $F(x) = \frac{3}{2}\cdot\ln(|(x^2+1)|)+C$

26) $f(x) = x\cdot e^{x^2}$; $F(x) = \alpha\cdot e^{x^2}$; $F'(x) = \alpha\cdot e^{x^2}\cdot 2x$; donc $\alpha = \frac{1}{2}$; $F(x) = \frac{1}{2}\cdot e^{x^2}+C$