

En utilisant le théorème $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$, calculez les limites suivantes (x en radians !):

1. $\lim_{x \rightarrow 0} \frac{\sin(3x)}{x}$	2. $\lim_{x \rightarrow 0} \frac{\sin(3x)}{2x}$
3. $\lim_{x \rightarrow 0} \frac{\sin(x/3)}{2x}$	4. $\lim_{x \rightarrow 0} \frac{\sin(ax)}{x}$
5. $\lim_{x \rightarrow 0} \frac{\tan(ax)}{x}$	6. $\lim_{x \rightarrow 0} \frac{\sin(2x)}{\sin(3x)}$
7. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x^2}$	8. $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x}$
9. $\lim_{x \rightarrow 0} \frac{\tan(x)}{x^2}$	10. $\lim_{x \rightarrow 0} \frac{\tan(5x)}{\sin(3x)}$
11. $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x-1}$	12. $\lim_{x \rightarrow \pi/2} \frac{\cos(x)}{x - \pi/2}$
13. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x^2 + x}$	14. $\lim_{x \rightarrow 0} \frac{2 \tan(x) + \sin(x)}{x}$
15. $\lim_{x \rightarrow 0} \frac{\sin(x^2 - 2x)}{x^2 - 2x}$	16. $\lim_{x \rightarrow 1} \frac{\sin(x^2 - 2x)}{x^2 - 2x}$
17. $\lim_{x \rightarrow 2} \frac{\sin(x^2 - 2x)}{x^2 - 2x}$	18. $\lim_{x \rightarrow 3} \frac{\sin(x^2 - 2x)}{x^2 - 2x}$
19. $\lim_{x \rightarrow a} \frac{\sin(x) - \sin(a)}{x - a}$	20. $\lim_{x \rightarrow a} \frac{\cos(x) - \cos(a)}{x - a}$
21. $\lim_{h \rightarrow 0} \frac{\sin(a+h) - \sin(a)}{h}$	22. $\lim_{h \rightarrow 0} \frac{\cos(a+h) - \cos(a)}{h}$

Les réponses se trouvent au dos...

Réponses, sans justifications !

1. $\lim_{x \rightarrow 0} \frac{\sin(3x)}{x} = 3$	2. $\lim_{x \rightarrow 0} \frac{\sin(3x)}{2x} = 3 / 2 = 1,5$
3. $\lim_{x \rightarrow 0} \frac{\sin(x/3)}{2x} = 1 / 6$	4. $\lim_{x \rightarrow 0} \frac{\sin(ax)}{x} = a$
5. $\lim_{x \rightarrow 0} \frac{\tan(ax)}{x} = a$	6. $\lim_{x \rightarrow 0} \frac{\sin(2x)}{\sin(3x)} = 2 / 3$
7. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x^2} = \pm\infty$, donc limite n'existe pas.	8. $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x} = 0$
9. $\lim_{x \rightarrow 0} \frac{\tan(x)}{x^2} = \pm\infty$, donc limite n'existe pas.	10. $\lim_{x \rightarrow 0} \frac{\tan(5x)}{\sin(3x)} = 5 / 3$
11. $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x-1} = 1$	12. $\lim_{x \rightarrow \pi/2} \frac{\cos(x)}{x - \pi/2} = -1$
13. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x^2 + x} = 1$	14. $\lim_{x \rightarrow 0} \frac{2 \tan(x) + \sin(x)}{x} = 3$
15. $\lim_{x \rightarrow 0} \frac{\sin(x^2 - 2x)}{x^2 - 2x} = 1$	16. $\lim_{x \rightarrow 1} \frac{\sin(x^2 - 2x)}{x^2 - 2x} = \sin(1)$, limite immédiate !
17. $\lim_{x \rightarrow 2} \frac{\sin(x^2 - 2x)}{x^2 - 2x} = 1$	18. $\lim_{x \rightarrow 3} \frac{\sin(x^2 - 2x)}{x^2 - 2x} = \sin(3) / 3$, limite immédiate !
19. $\lim_{x \rightarrow a} \frac{\sin(x) - \sin(a)}{x - a} = \cos(a)$	20. $\lim_{x \rightarrow a} \frac{\cos(x) - \cos(a)}{x - a} = -\sin(a)$
21. $\lim_{h \rightarrow 0} \frac{\sin(a+h) - \sin(a)}{h} = \cos(a)$	22. $\lim_{h \rightarrow 0} \frac{\cos(a+h) - \cos(a)}{h} = -\sin(a)$