Limites

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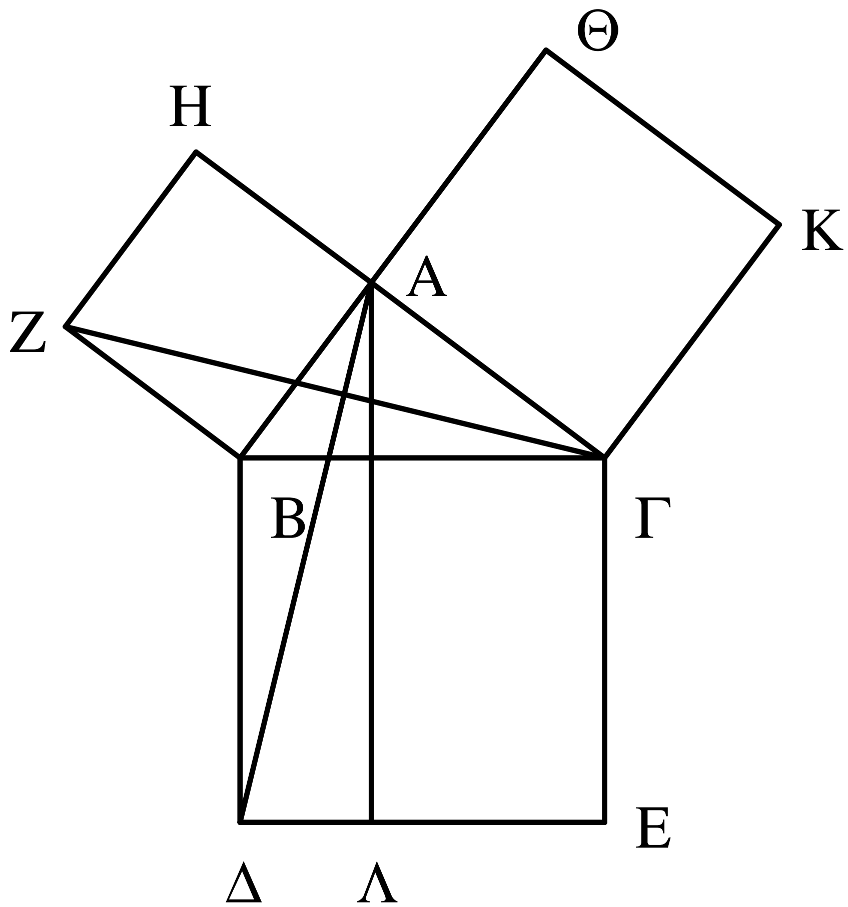
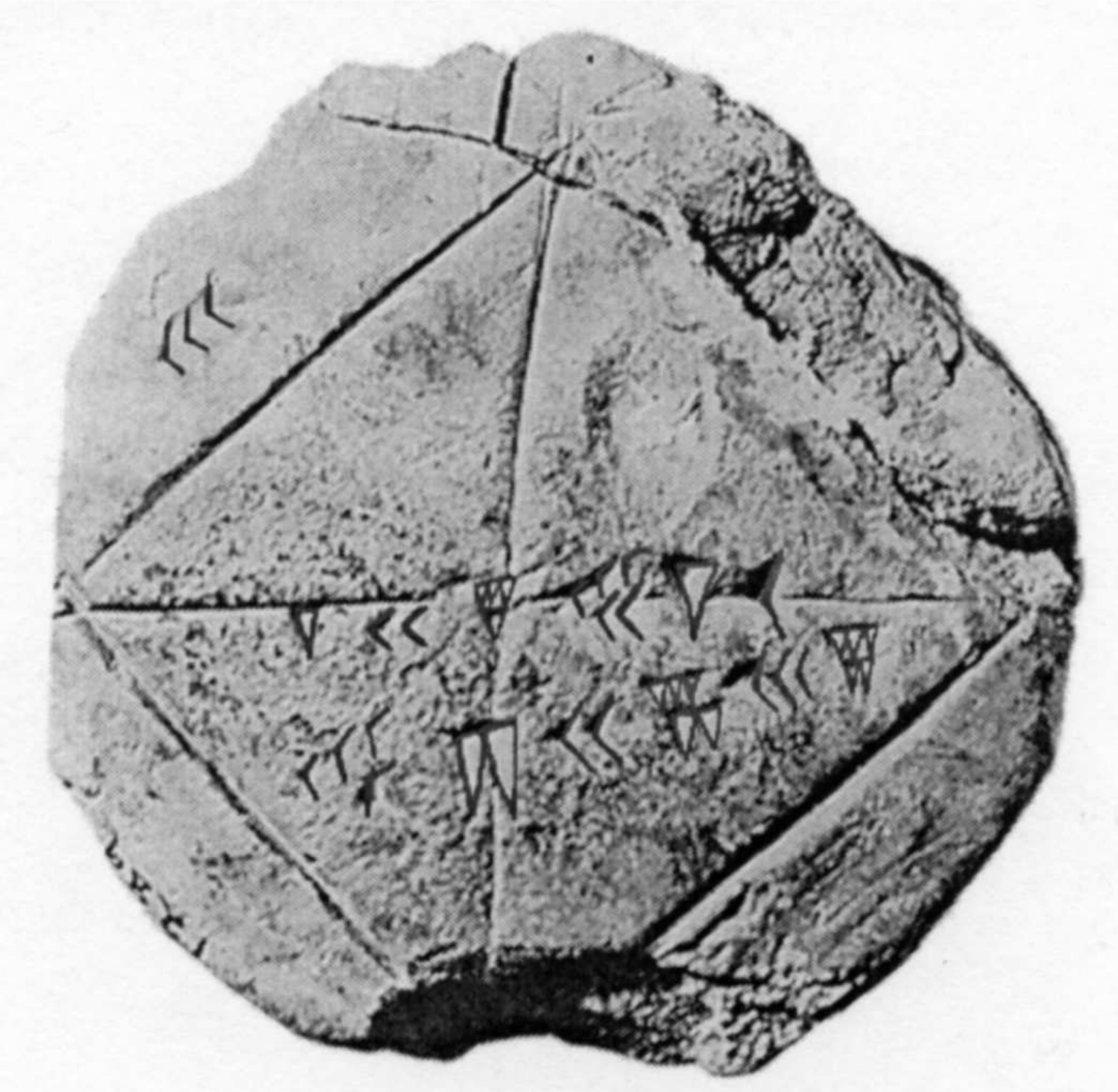
“Die Entwicklung der abendländischen Naturwissenschaften beruht auf zwei grossen Leistungen: Der Erfindung des formal logischen Systems (in der euklidischen Geometrie) durch die griechischen Philosophen, und die Entdeckung der Möglichkeit, nach systematischen Experimenten kausale Beziehungen herzustellen.”

(A. Einstein, 1953)

Géométrie classique



Babylon, 1900 av.J.-C.



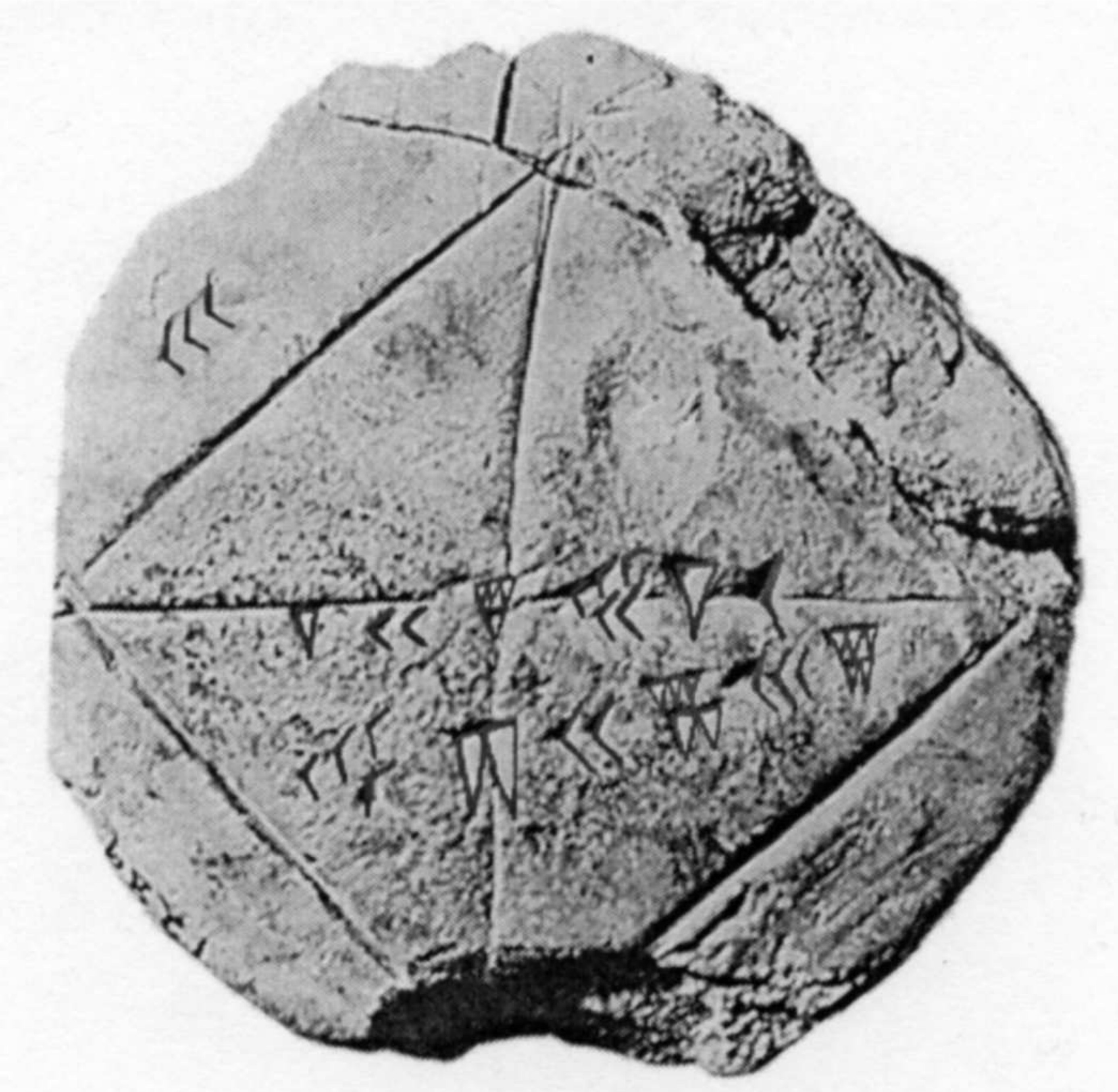
Babylon, 1900 av.J.-C.

30

1, 24 51 10

42, 25 35

Babylon, 1900 av.J.-C.



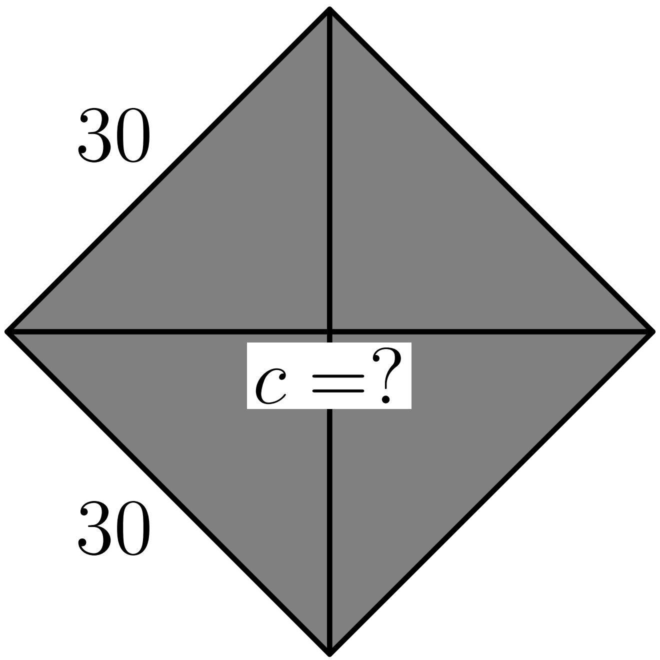
30

1, 24 51 10

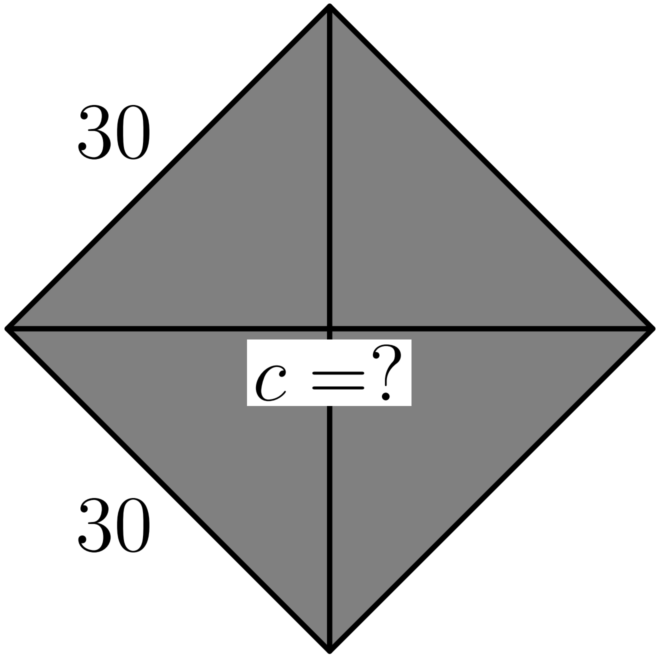
42, 25 35

comprendre ??

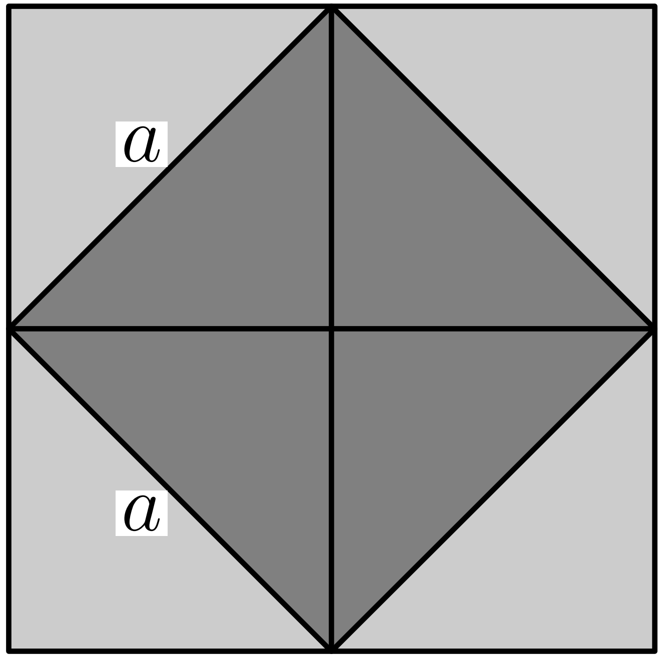
Problème:



Problème:



Solution:



$$c^2 = 2a^2$$

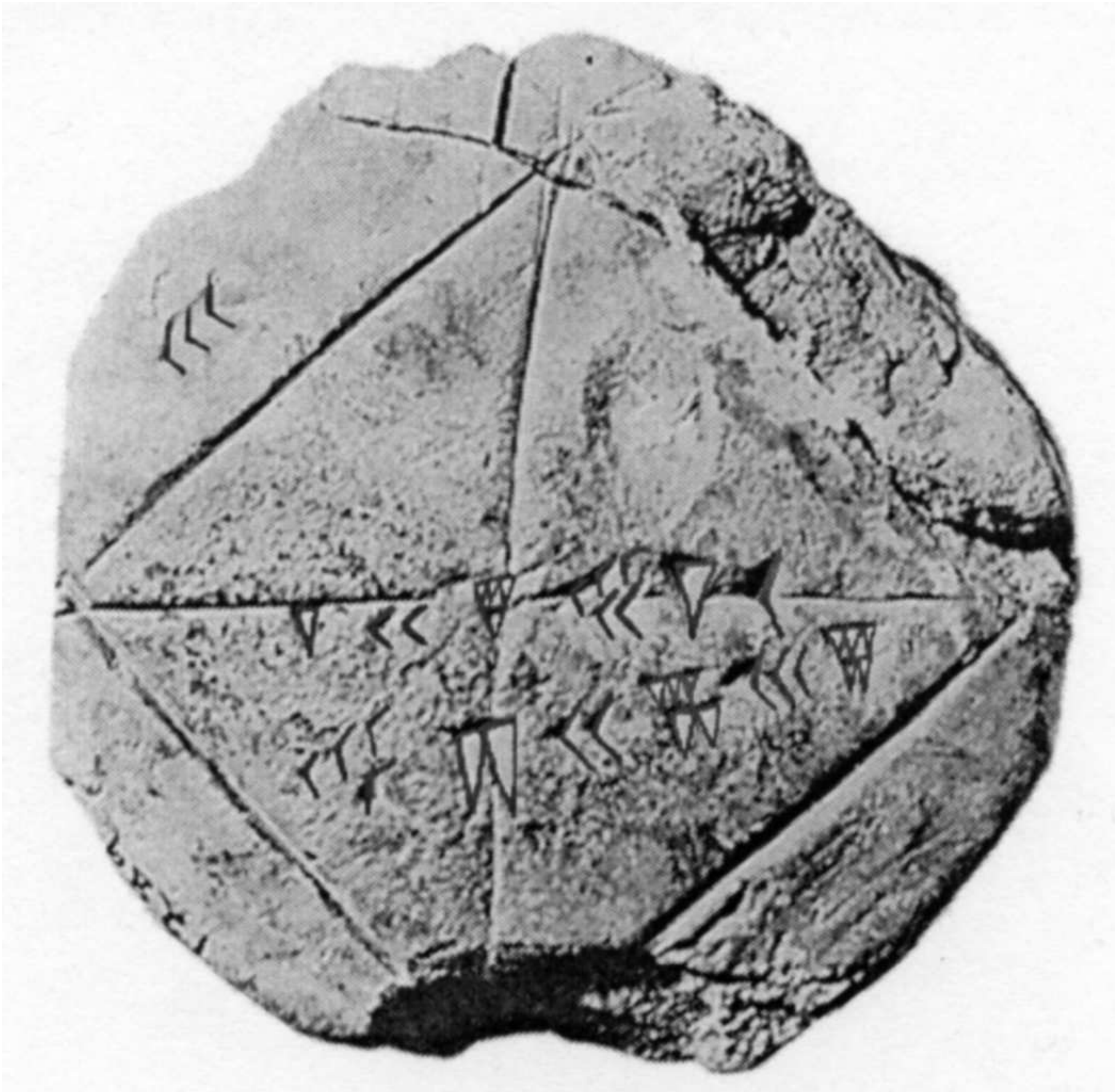
$$\Rightarrow \qquad c = a\sqrt{2}$$

$$\sqrt{2} = 1.4142135623730950488016887242096939\ldots$$

$$a = 30 \quad \Rightarrow \quad c = 42.426406871192851464050661726290670\ldots$$

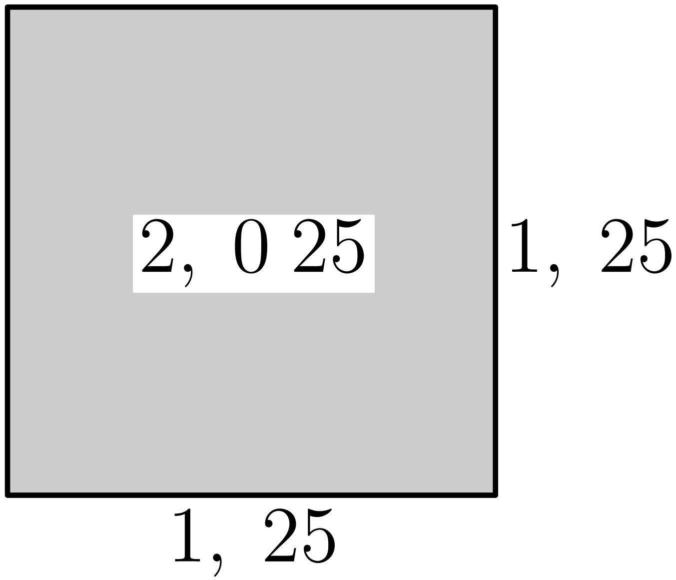
Eh bien, en base 60, nous avons les valeurs exactes

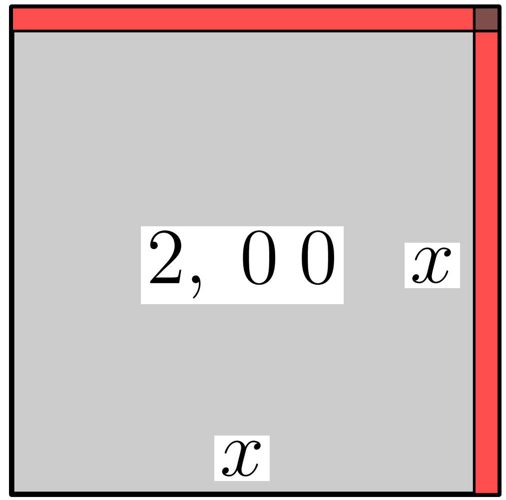
$\sqrt{2} = 1,\ 24\,51\,10\,7\,46\,6\,4\ldots$ $\qquad$ $30{\cdot}\sqrt{2} = 42,\ 25\,35\,3\,53\,3\,2\ldots$



Comment les Babyloniens ont trouvé (probablement) cette valeur?

1	22	1	52	4
1	23	1	54	49
1	24	1	57	36
1	25	2	0	25
1	26	2	3	16
1	27	2	6	9
1	28	2	9	4

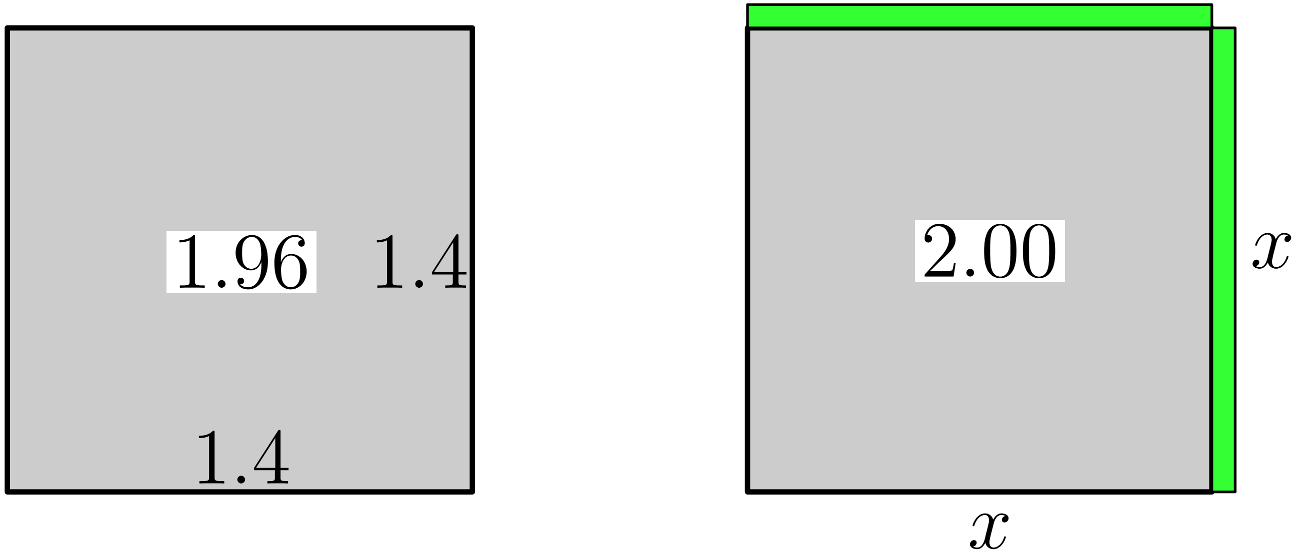




$$x = 1,\ 25 \ - \ \frac{0,\ 0\,25}{2 \cdot (1,\ 25)} \ = \ 1,\ 24\,51\,10\ldots$$

la “méthode babylonienne pour calculer la racine”.

La même chose en base 10:



$$x = 1.4 + \frac{0.04}{2 \cdot 1.4} = 1.4 + 0.01428... = 1.41428...$$

1.414285 répéter :

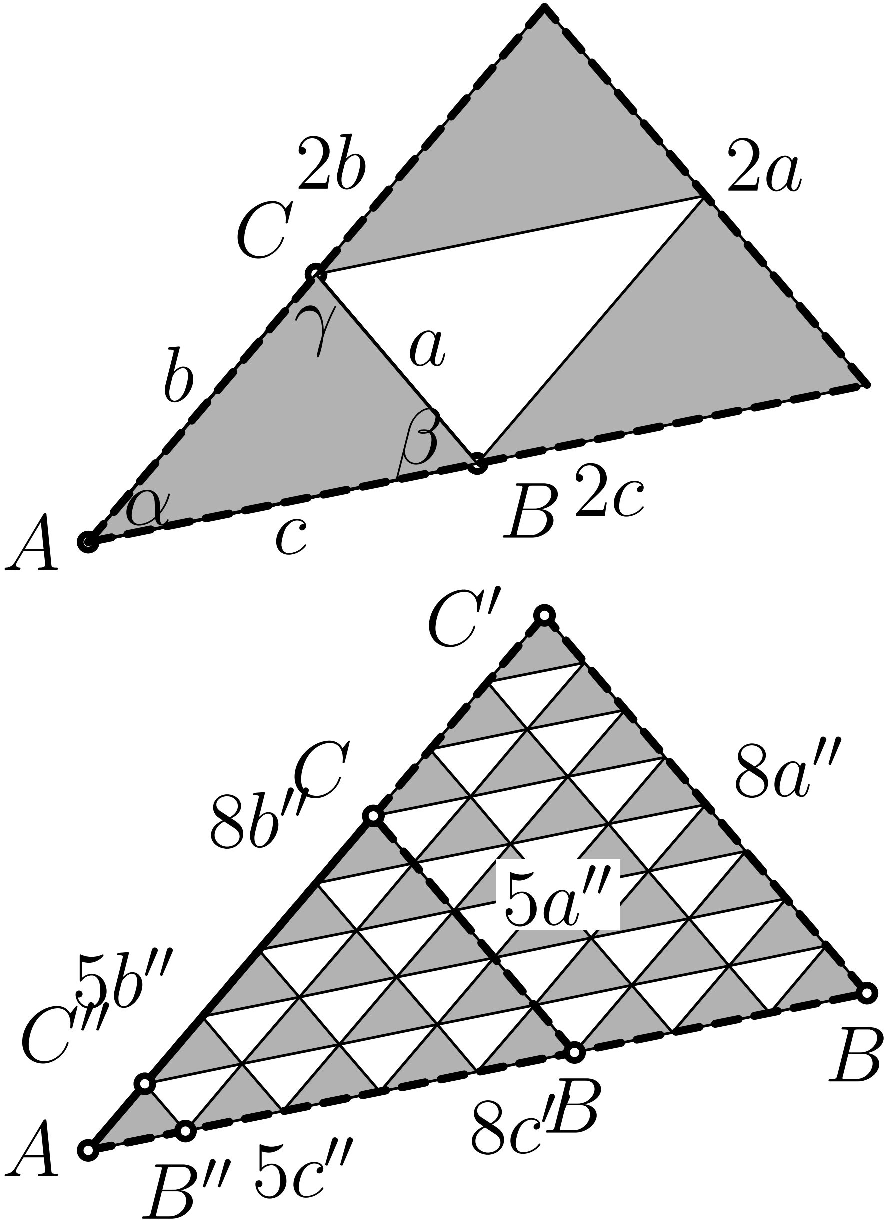
1.4142135642

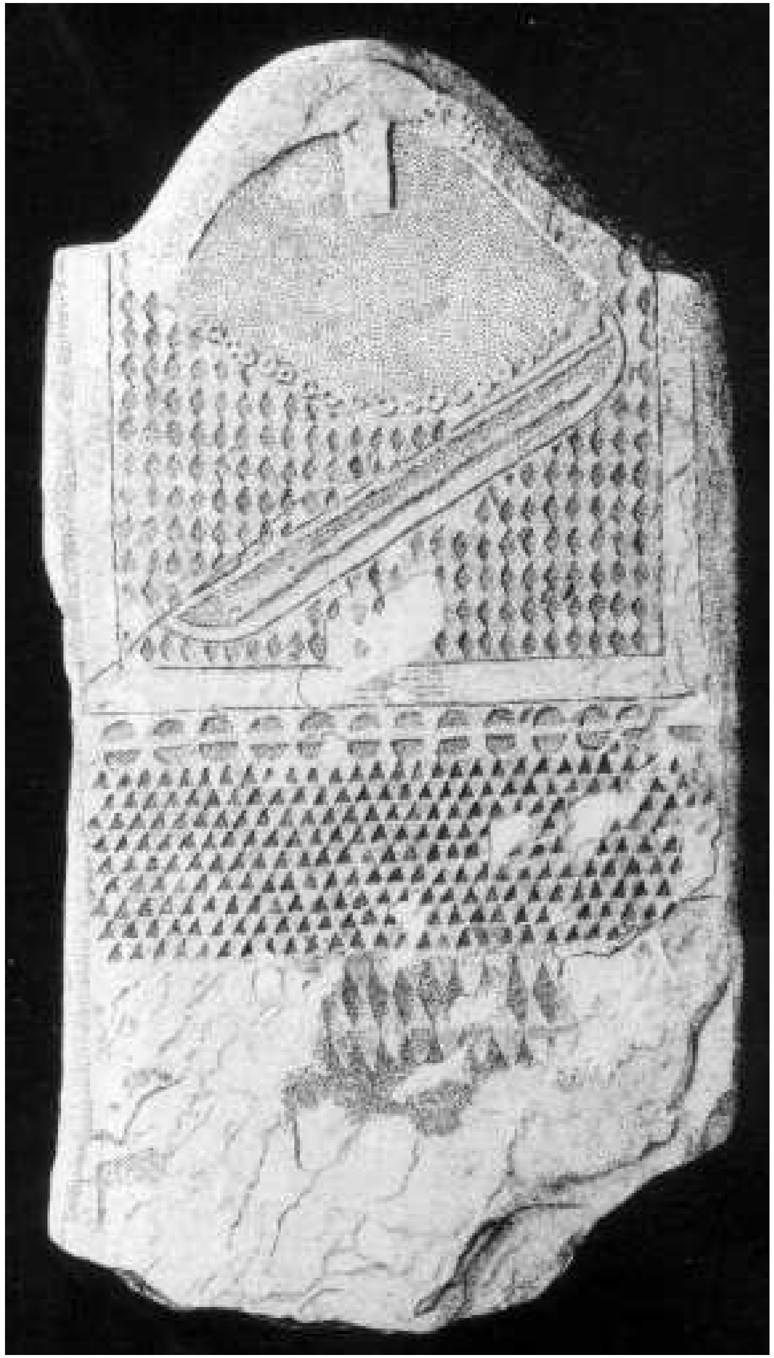
1.4142135623730950499

1.4142135623730950488016887242096980790

1.4142135623730950488016887242096980785696718753769480

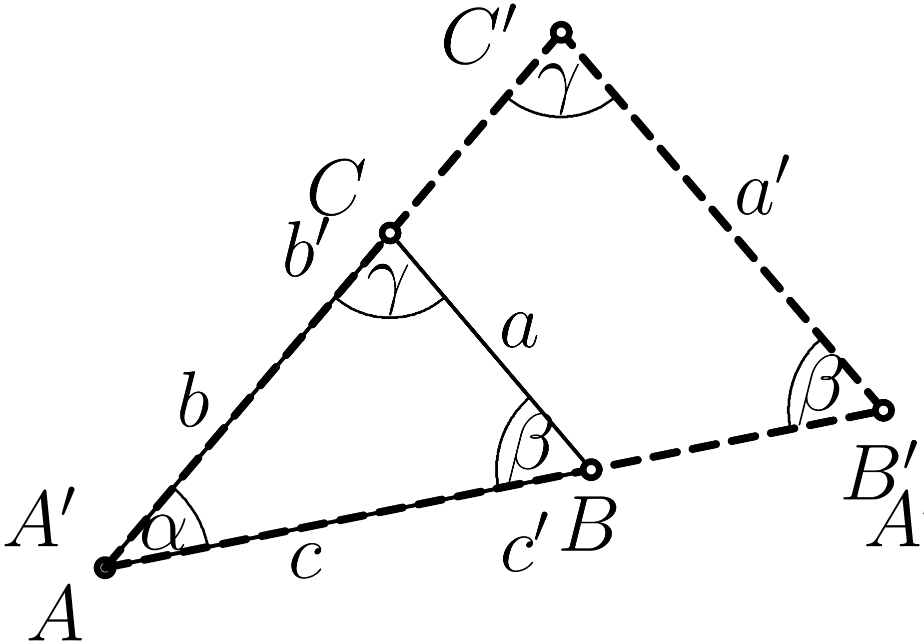
Théorème de Thales

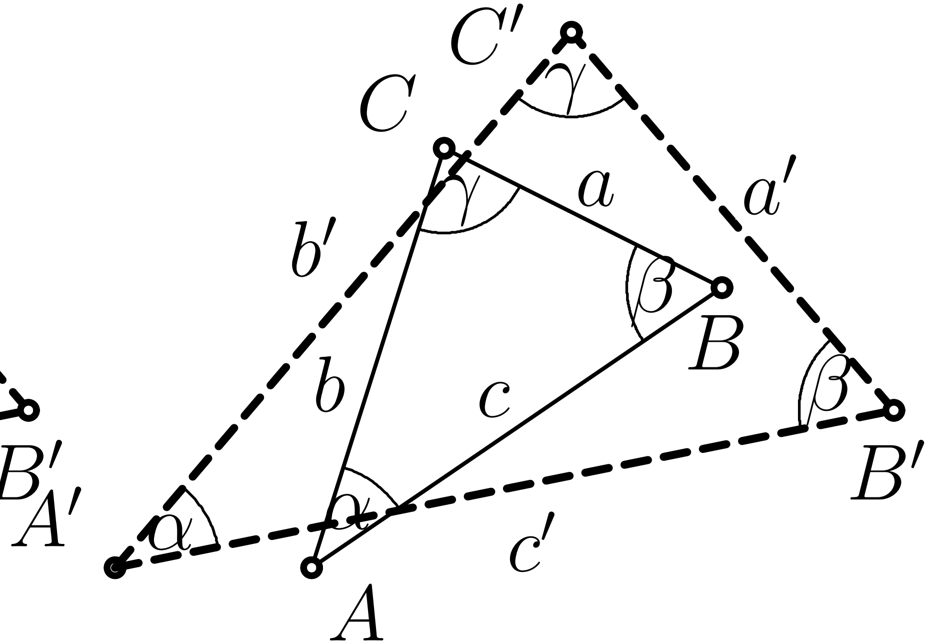




Thales. "... la théorie des lignes proportionnelles et la proposition de Pythagore, qui sont les bases de la Géométrie ..."

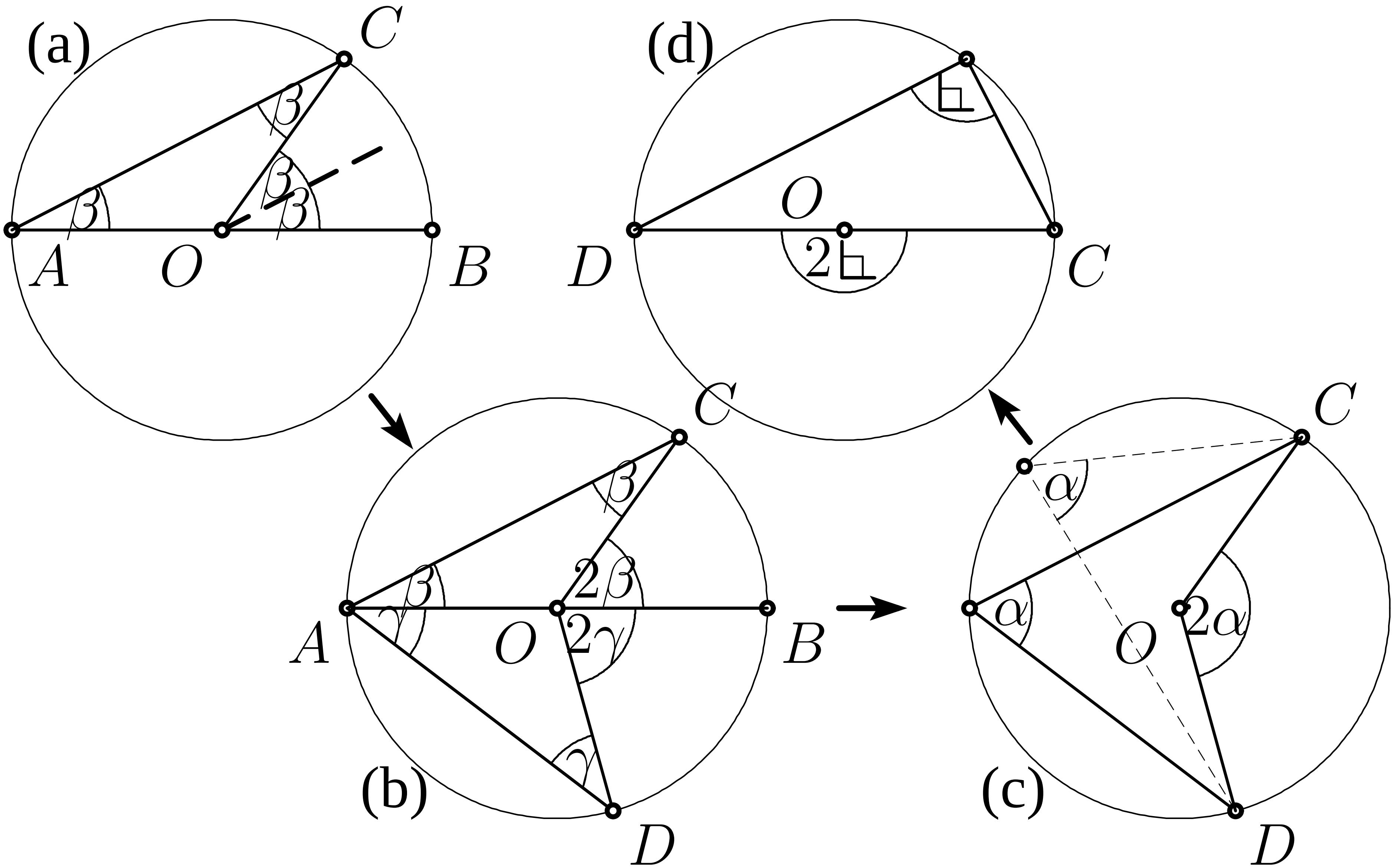
(Poncelet 1822, p. xxix)



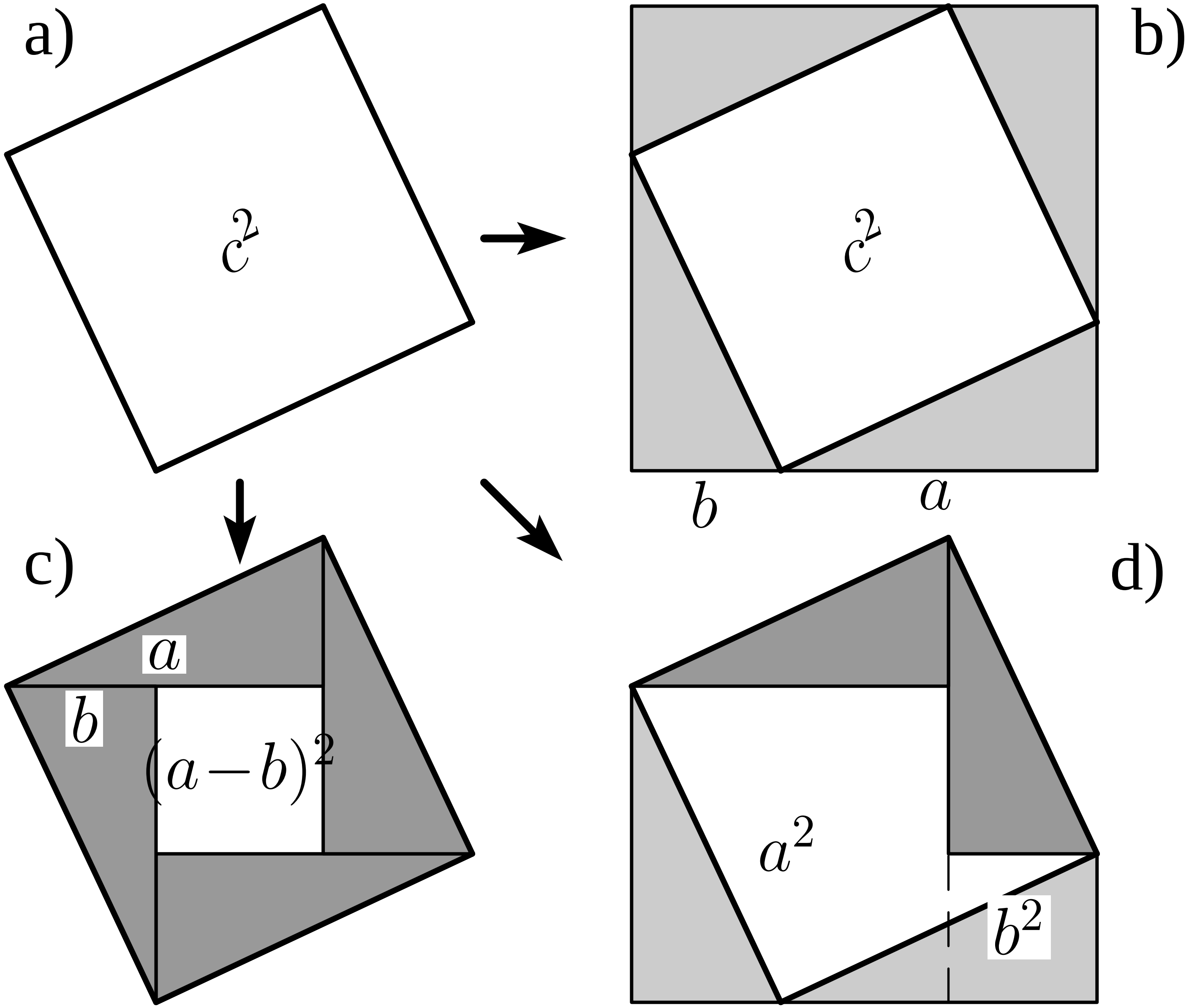


$$\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c} \qquad \text{oder} \qquad \frac{a'}{c'} = \frac{a}{c}\,, \qquad \frac{c'}{b'} = \frac{c}{b}\,, \qquad \frac{b'}{a'} = \frac{b}{a}\,.$$

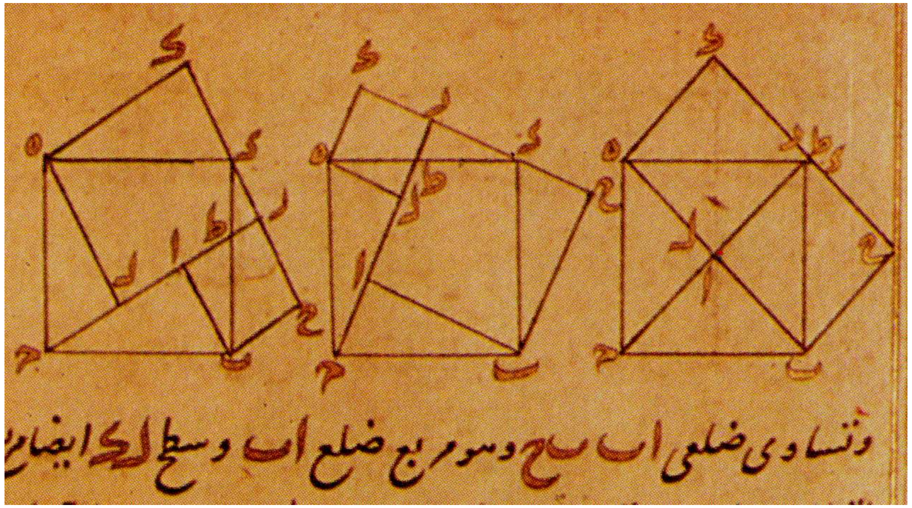
Théorème de l'angle au centre – angle périphérique.

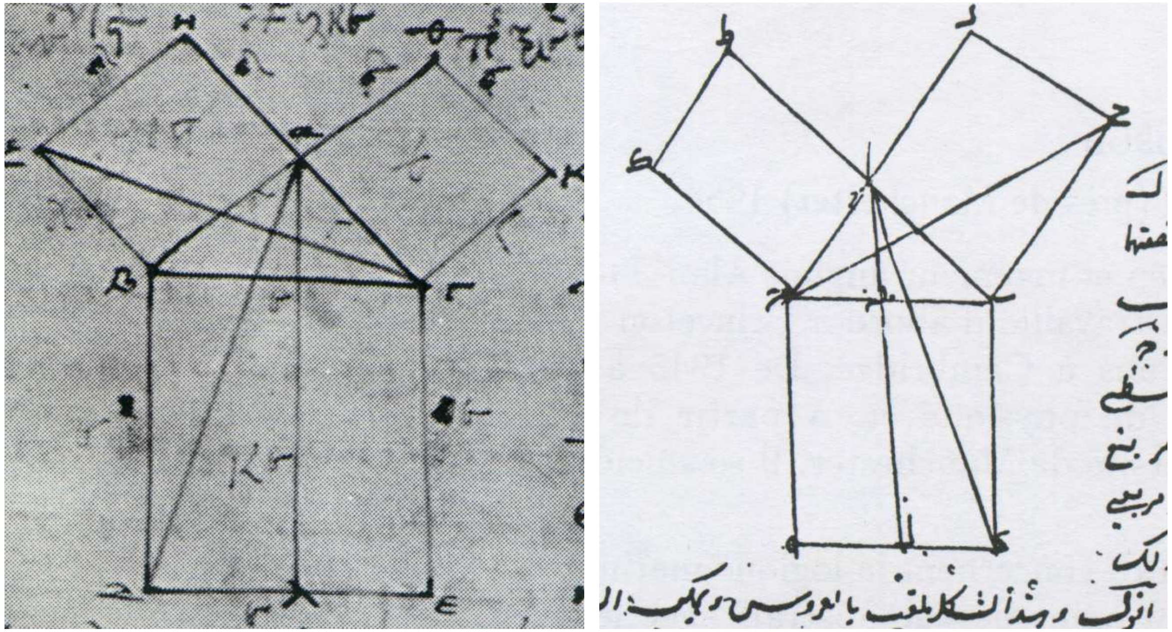


Théorème de Pythagore.



Th. Pythagore.

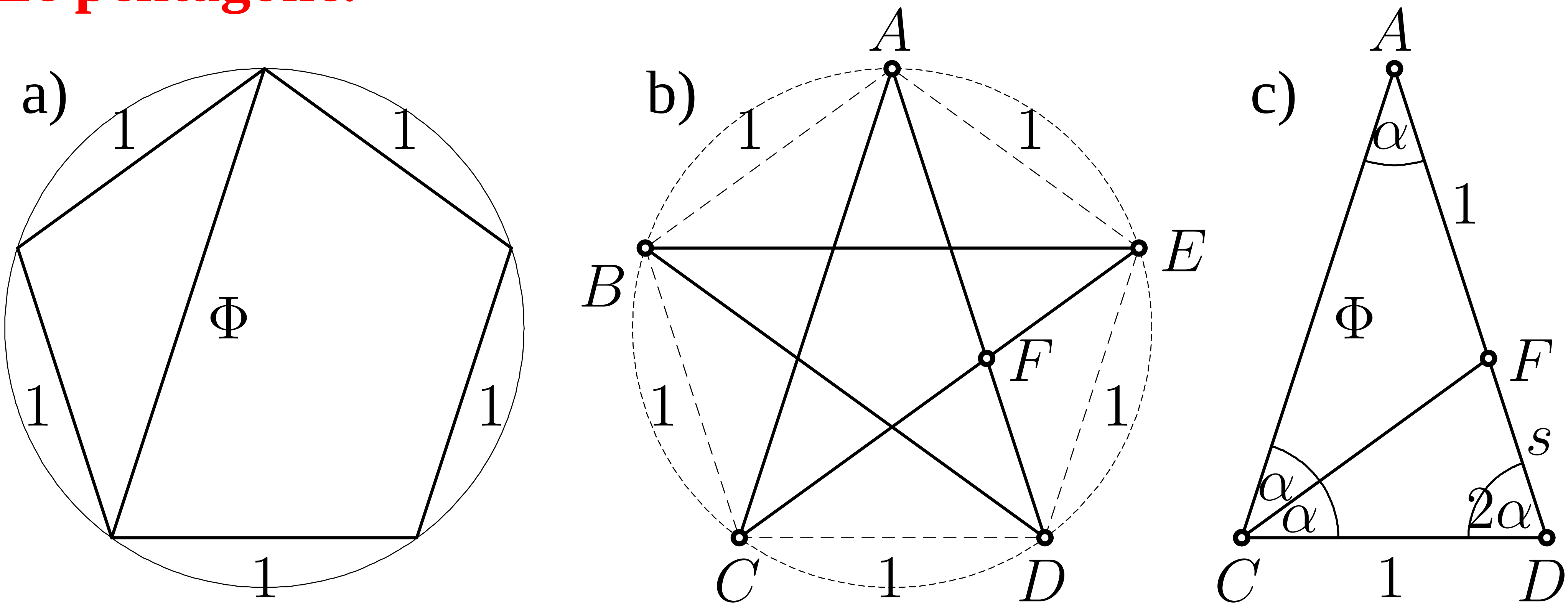




Thābit Ibn Qurra, 870 n. Chr.

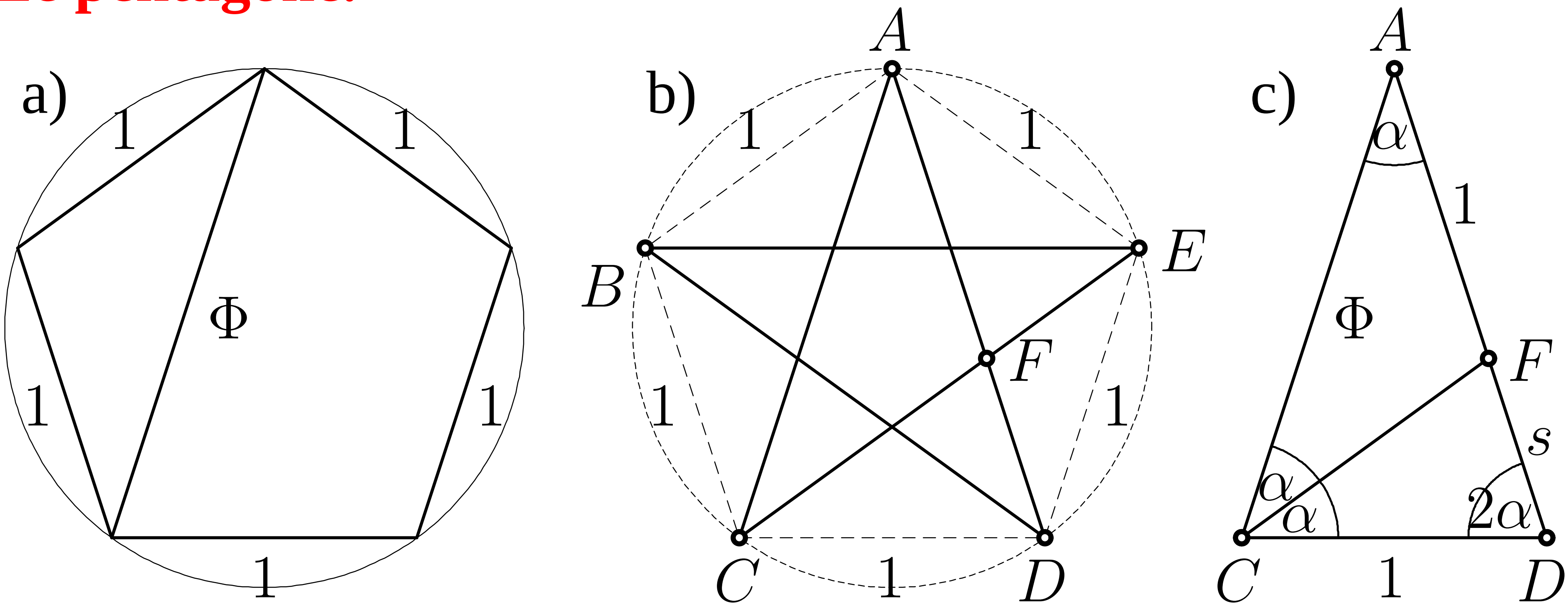
Euklid 300 v. Chr.

Le pentagone.



$$\Phi = 1 + \frac{1}{\Phi} = \frac{1+\sqrt{5}}{2}$$

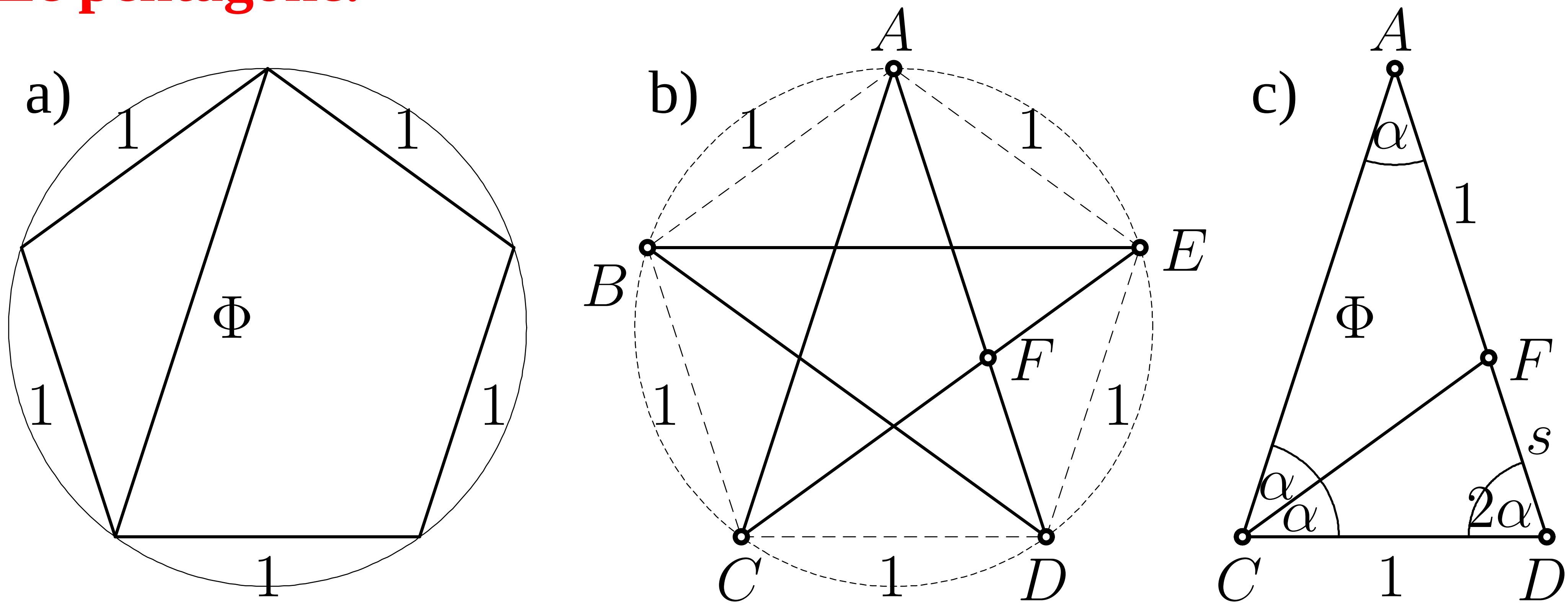
Le pentagone.



$$\Phi = 1 + \frac{1}{\Phi} = \frac{1+\sqrt{5}}{2} \quad \Rightarrow \quad \frac{m}{n} = 1 + \frac{1}{\frac{m}{n}} = 1 + \frac{n}{m} = \frac{m+n}{m}\,.$$

n’est pas rationnel !!

Le pentagone.



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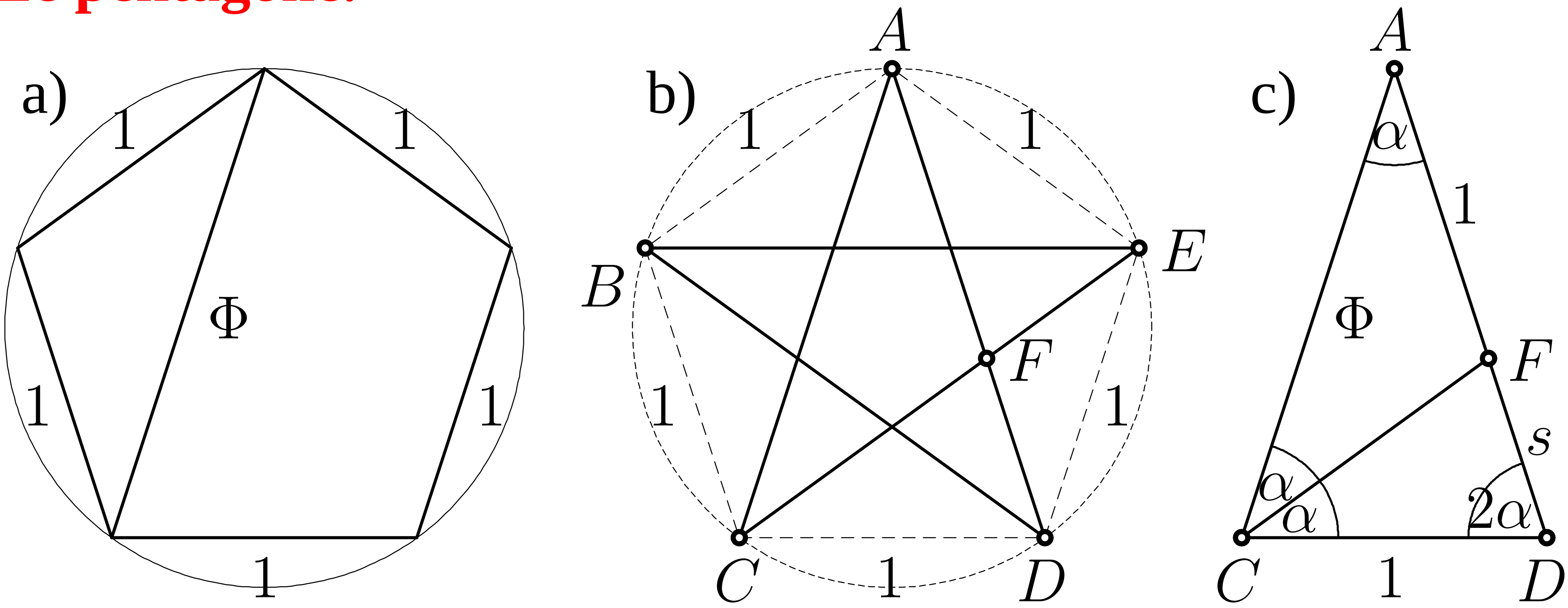
n'est pas rationnel !!

Gros choc : les preuves sont insuffisantes !!!

Le pentagone.



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n'est pas rationnel !!

Gros choc : les preuves sont insuffisantes !!!

... et $\sqrt{2}$ n'est pas rationnel non plus !!

Sauvetage : **LA LIMITE !**

$\sqrt{2}$ est la limite de $1.4,\ 1.414285,\ 1.4142135642, \ldots$

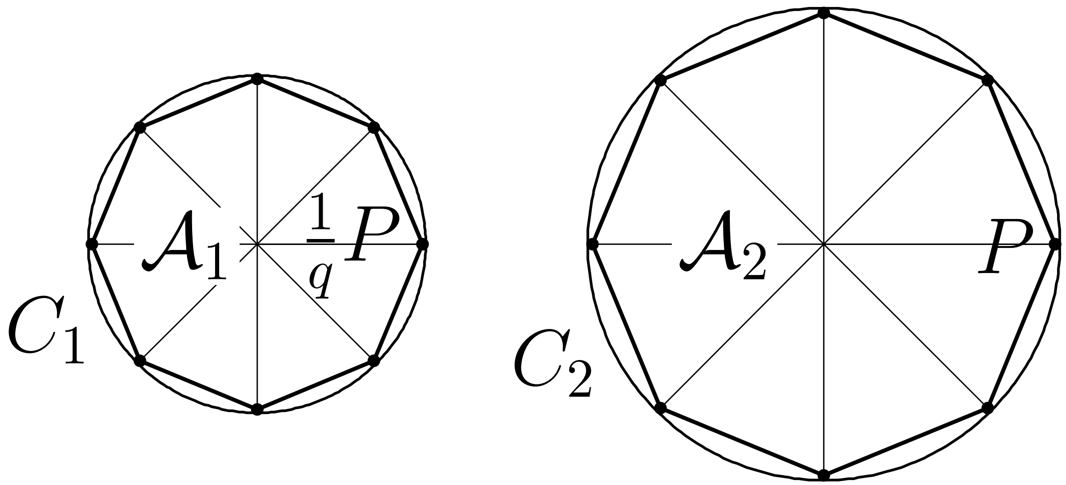
Φ est la limite de $\frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \ldots$

triangle à rapports irrationels est la limite de triangles rationaux.

Euclide, Livre XII. Aires et volumes de cercles, pyramides, cônes et sphères.

Eucl. XII.2. *L'aire $\mathcal{A}$ d'un cercle C de rayon r est*

$$\mathcal{A} = r^2 \cdot \pi \qquad (\pi = \text{l'aire du cercle de rayon } 1.)$$



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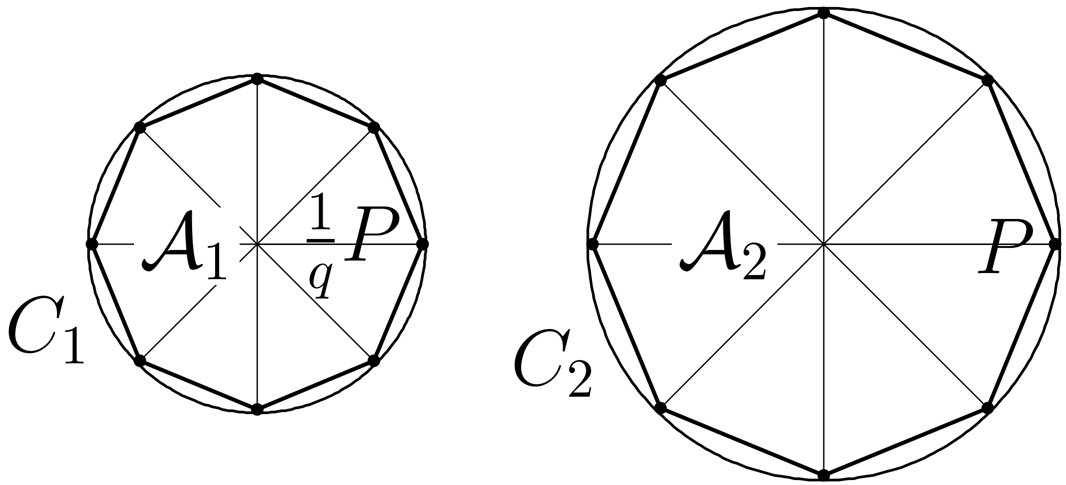

Archimedes : π est la limite des aires du 6-gone, 12-gone, 24-gone, 48-gone, 96-gone . . .

$$\pi = 3\frac{1}{7}$$

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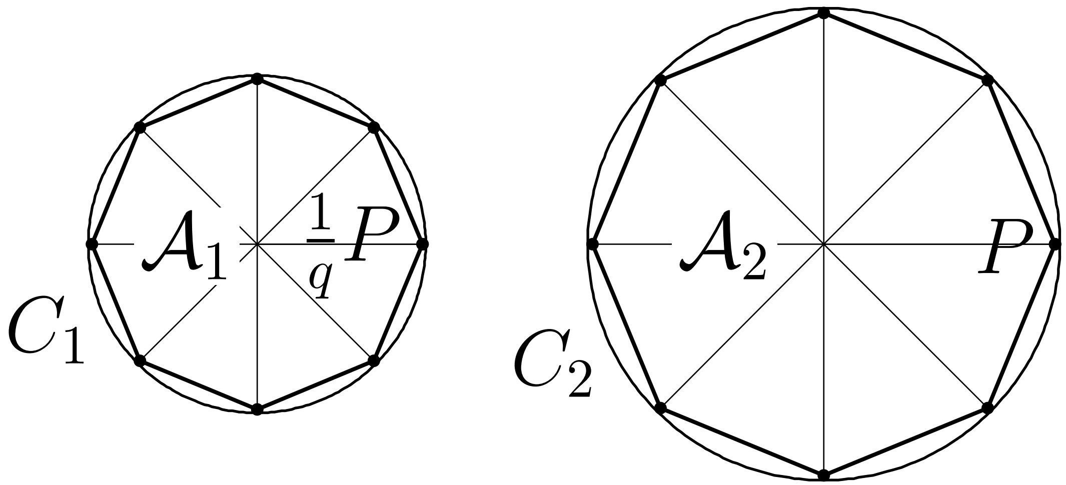
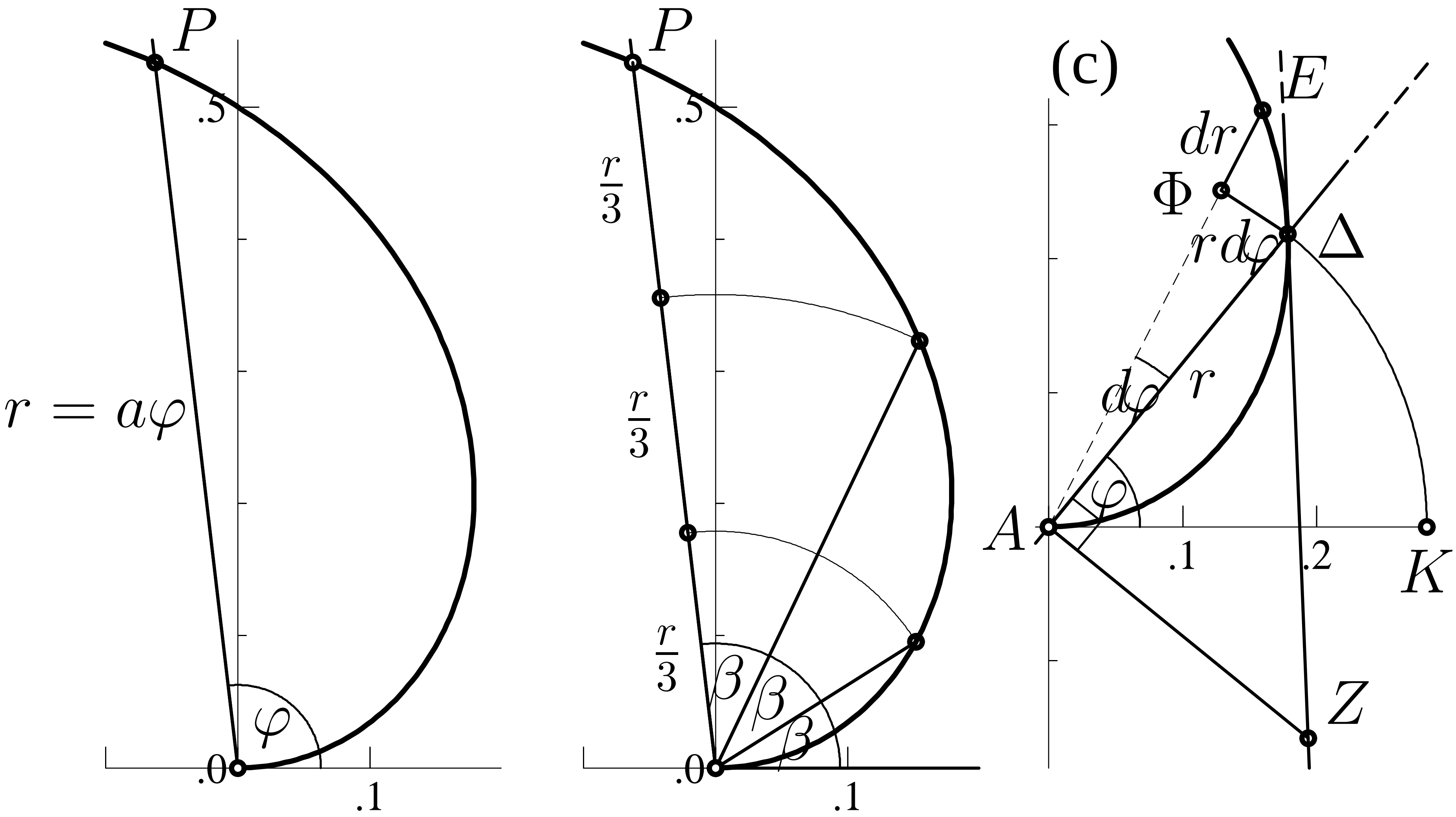


Archimedes : π est la limite des aires du 6-gone, 12-gone, 24-gone, 48-gone, 96-gone ...

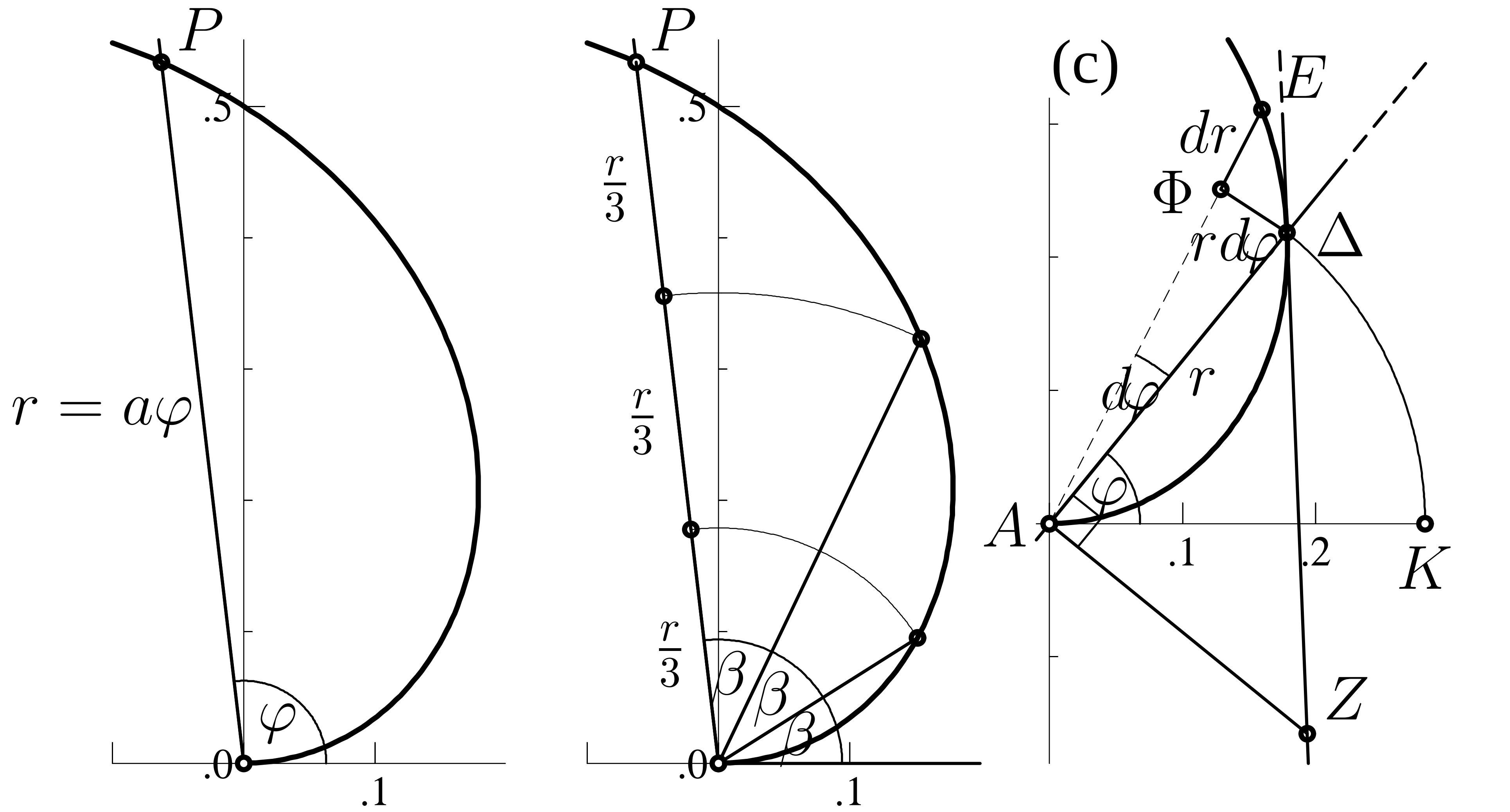
$\pi = 3\frac{1}{7}$ Ludolph 1596: $3.1415926535897932384626433832795028$

La spirale d'Archimède.



Archimedes : Tangente est la limite pour $d\varphi \to 0$.

La spirale d'Archimède.



Archimedes : Tangente est la limite pour $d\varphi \to 0$.

Attention, historiens!! :

Notation dx, dy : Leibniz 1675, notation lim : Cauchy 1821.

... Et 2000 ans plus tard: la science d’aujourd’hui :