❶ Calcul, de dérivées :

a) $\left(3x^6-11x^3+x^2-8x-3\right)'=18x^5-33x^2+2x-8$
b) $\left(-5x^8+7x^4-19x^3+4x+12\right)'=-40x^7+28x^3-57x^2+4$
c) $\left(3x^6-11x^3+x^2-3-\frac{8}{x}\right)'=18x^5-33x^2+2x+\frac{8}{x^2}$
d) $\left(\frac{1}{x^2}+x\cdot\sqrt{x}\right)'=-2\cdot x^{-3}+\left(x^{\frac{3}{2}}\right)'=\frac{-2}{x^3}+\frac{3}{2}\cdot x^{\frac{1}{2}}=\frac{-2}{x^3}+\frac{3}{2}\cdot\sqrt{x}$
e) $\left(2x^7+4x^2+1-\frac{5}{x^2}+\frac{1}{x^3}\right)'=14x^6+8x-5\cdot\left(x^{-2}\right)'+\left(x^{-3}\right)'=14x^6+8x+10x^{-3}-3x^{-4}$
f) $\left(x^2\cdot\sqrt[5]{x^3}\right)'=\left(x^{2+\frac{3}{5}}\right)'=\left(x^{\frac{13}{5}}\right)'=\frac{13}{5}\cdot x^{\frac{8}{5}}$
g) $\left(\frac{3x^2+2x^6}{6x^4}\right)'=\left(\frac{3x^2}{6x^4}+\frac{2x^6}{6x^4}\right)'=\left(\frac{1}{2}\cdot x^{-2}+\frac{1}{3}\cdot x^2\right)'=-x^{-3}+\frac{2}{3}x=\frac{-1}{x^3}+\frac{2}{3}x$
h) $\left(x^2\cdot\sin(x)\right)'=2x\cdot\sin(x)+x^2\cdot\cos(x)=x\cdot\left[2\cdot\sin(x)+x\cdot\cos(x)\right]$
i) $\left(\frac{2x^4+x}{x^3}\right)'=\left(\frac{2x^4}{x^3}+\frac{x}{x^3}\right)'=\left(2x+\frac{1}{x^2}\right)'=2-2\cdot x^{-3}=2-\frac{2}{x^3}$
j) $\left(\frac{3x+1}{x-2}\right)'=\frac{3\cdot(x-2)-(3x+1)\cdot 1}{(x-2)^2}=\frac{-7}{(x-2)^2}$
k) $\left(2\cdot\left(x^3+4\right)^5\right)'=2\cdot\left(\left(x^3+4\right)^5\right)'=2\cdot5\cdot\left(x^3+4\right)^4\cdot3x^2=30x^2\cdot\left(x^3+4\right)^4$
l) $\left(x\cdot\sin^2(x)\right)'=1\cdot\sin^2(x)+x\cdot2\cdot\sin(x)\cdot\cos(x)=\sin(x)\cdot\left[\sin(x)+2x\cdot\cos(x)\right]$
m) $\left(4\cdot\cos^3(x)\right)'=4\cdot\left(\cos^3(x)\right)'=4\cdot3\cdot\cos^2(x)\cdot\left[-\sin(x)\right]=-12\cdot\sin(x)\cdot\cos^2(x)$
n) $\left(\left(\frac{3x-1}{2x+3}\right)^3\right)'=3\cdot\left(\frac{3x-1}{2x+3}\right)^2\cdot\frac{3\cdot(2x+3)-(3x-1)\cdot2}{(2x+3)^2}=3\cdot\left(\frac{3x-1}{2x+3}\right)^2\cdot\frac{11}{(2x+3)^2}=\frac{33\cdot(3x-1)^2}{(2x+3)^4}$
o) $\left(\frac{\cos(x)}{x}\right)'=\frac{-\sin(x)\cdot x-\cos(x)\cdot1}{x^2}=-\frac{x\cdot\sin(x)+\cos(x)}{x^2}$
p) $\left((1+x)\cdot\sin^2(x)\right)'=1\cdot\sin^2(x)+(1+x)\cdot2\cdot\sin(x)\cdot\cos(x)=\sin^2(x)+(1+x)\cdot2\cdot\sin(x)\cdot\cos(x)$
q) $\left(x^3\cdot\left[x+\cos(x)\right]\right)'=3x^2\cdot\left[x+\cos(x)\right]+x^3\cdot\left[1-\sin(x)\right]=x^2\cdot\left[3x+3\cdot\cos(x)+x-x\cdot\sin(x)\right]=$ $x^2\cdot\left[4x+3\cdot\cos(x)-x\cdot\sin(x)\right]$

r) $\left(x^2\cdot\sin(x)\right)' = 2x\cdot\sin(x)+x^2\cdot\cos(x) = x\cdot\left[2\cdot\sin(x)+x\cdot\cos(x)\right]$

s) $\left(\sin^2(x)+\cos^2(x)\right)' = [1]' = 0$

t) $\left(\sin^2(x)-\cos^2(x)\right)' = 2\cdot\sin(x)\cdot\cos(x)-2\cdot\cos(x)\cdot\left[-\sin(x)\right] = 4\cdot\sin(x)\cdot\cos(x)$

❷

a) $$\left(\frac{(2x-3)^3}{(5-x)^2}\right)' = \frac{3\cdot(2x-3)^2\cdot 2\cdot(5-x)^2-(2x-3)^3\cdot 2\cdot(5-x)\cdot(-1)}{(5-x)^4} =$$
$$\frac{(2x-3)^2\cdot(5-x)\cdot\left[6\cdot(5-x)+2\cdot(2x-3)\right]}{(5-x)^4} = \frac{(2x-3)^2\cdot(5-x)\cdot\left[-2x+24\right]}{(5-x)^4} = \frac{2\cdot(2x-3)^2\cdot\left[12-x\right]}{(5-x)^3}$$

b) $$\left(e^{\cos\left(x^2\right)}\right)' = e^{\cos\left(x^2\right)}\cdot\left[-\sin(x^2)\right]\cdot 2x = -2\cdot x\cdot\sin(x^2)\cdot e^{\cos\left(x^2\right)}$$

c) $$\left(\sqrt{\frac{2x-3}{x+2}}\right)' = \frac{1}{2}\cdot\left(\frac{2x-3}{x+2}\right)^{-\frac{1}{2}}\cdot\frac{2\cdot(x+2)-(2x-3)\cdot 1}{(x+2)^2} = \frac{1}{2}\cdot\frac{(x+2)^{\frac{1}{2}}}{(2x-3)^{\frac{1}{2}}}\cdot\frac{7}{(x+2)^2} = \frac{7}{2}\cdot\frac{1}{(2x-3)^{\frac{1}{2}}\cdot(x+2)^{\frac{3}{2}}} =$$
$$\frac{7}{2\cdot(x+2)\cdot\sqrt{(2x-3)\cdot(x+2)}}$$

d) $$\left(\ln\left(\left(2x^2+5x-3\right)^4\right)\right)' = \frac{1}{\left(2x^2+5x-3\right)^4}\cdot 4\cdot\left(2x^2+5x-3\right)^3\cdot(4x+5) = \frac{4\cdot(4x+5)}{\left(2x^2+5x-3\right)}$$

Autre manière : $$\left(\ln\left(\left(2x^2+5x-3\right)^4\right)\right)' = 4\cdot\left(\ln\left(2x^2+5x-3\right)\right)' = \frac{4}{2x^2+5x-3}\cdot(4x+5) = \frac{4\cdot(4x+5)}{2x^2+5x-3}$$

e) $$\left(e^{\sin\left(x^2\right)}\right)' = e^{\sin\left(x^2\right)}\cdot\cos\left(x^2\right)\cdot 2x = 2x\cdot\cos\left(x^2\right)\cdot e^{\sin\left(x^2\right)}$$

f) $$\left(\frac{e^x}{1+x^2}\right)' = \frac{e^x\cdot\left(1+x^2\right)-e^x\cdot 2x}{\left(1+x^2\right)^2} = \frac{e^x\cdot\left(1+x^2-2x\right)}{\left(1+x^2\right)^2} = \frac{(1-x)^2\cdot e^x}{\left(1+x^2\right)^2}$$

g) $$\left(\ln\left(x^2\cdot e^x\right)\right)' = \frac{1}{x^2\cdot e^x}\cdot\left[2x\cdot e^x+x^2\cdot e^x\right] = \frac{x\cdot e^x\cdot\left[2+x\right]}{x^2\cdot e^x} = \frac{\left[2+x\right]}{x} = \frac{2}{x}+1$$

Autre manière : $$\left(\ln\left(x^2\cdot e^x\right)\right)' = \left(\ln\left(x^2\right)+\ln\left(e^x\right)\right)' = \left(2\cdot\ln(x)+x\right)' = \frac{2}{x}+1$$

h) $$\left(x\cdot e^{-\frac{1}{x}}\right)' = 1\cdot e^{-\frac{1}{x}}+x\cdot e^{-\frac{1}{x}}\cdot\frac{1}{x^2} = e^{-\frac{1}{x}}\cdot\left(1+\frac{1}{x}\right)$$