

❶ Calculez les dérivées des fonctions suivantes :

1.1)	$[\sin(2x)]' = 2 \cdot \cos(2x)$	1.2)	$\left[\cos\left(\frac{x}{3}\right)\right]' = -\frac{1}{3} \cdot \sin\left(\frac{x}{3}\right)$
1.3)	$[\sin(1+x^2)]' = 2x \cdot \cos(1+x^2)$	1.4)	$[\tan(3x)]' = \frac{3}{\cos^2(3x)} = \underline{\underline{3 \cdot (1 + \tan^2(3x))}}$

1.5)	$[\sin^3(4x)]' = 3 \cdot \sin^2(4x) \cdot \cos(4x) \cdot 4 = 12 \cdot \sin^2(4x) \cdot \cos(4x)$
1.6)	$[3 \cdot \cos(x^2 - 1)]' = 3 \cdot [-\sin(x^2 - 1)] \cdot 2x = -6x \cdot \sin(x^2 - 1)$
1.7)	$[\sqrt{2x+1}]' = \left[(2x+1)^{\frac{1}{2}}\right]' = \frac{1}{2} \cdot (2x+1)^{-\frac{1}{2}} \cdot 2 = \frac{1}{\sqrt{2x+1}}$
1.8)	$[3 \cdot \sqrt{x^2 + 4x - 5}]' = 3 \cdot \left[(x^2 + 4x - 5)^{\frac{1}{2}}\right]' = 3 \cdot \frac{1}{2} \cdot (x^2 + 4x - 5)^{-\frac{1}{2}} \cdot (2x + 4) = \frac{3 \cdot 2 \cdot (x + 2)}{2} \cdot \frac{1}{\sqrt{x^2 + 4x - 5}}$ $= \frac{3 \cdot (x + 2)}{\sqrt{x^2 + 4x - 5}}$
1.9)	$\left[\frac{1}{\sqrt{3x^2 - 1}}\right]' = \left[(3x^2 - 1)^{-\frac{1}{2}}\right]' = \frac{-1}{2} \cdot (3x^2 - 1)^{-\frac{3}{2}} \cdot 6x = \frac{-3x}{\sqrt{(3x^2 - 1)^3}} = \frac{-3x}{(3x^2 - 1) \cdot \sqrt{(3x^2 - 1)}}$
1.10)	$\left[\frac{8}{\sqrt{8x^2 + 2}}\right]' = 8 \cdot \left[(8x^2 + 2)^{-\frac{1}{2}}\right]' = 8 \cdot \frac{-1}{2} \cdot (8x^2 + 2)^{-\frac{3}{2}} \cdot 16x = -64x \cdot \frac{1}{\sqrt{(8x^2 + 2)^3}}$ $\frac{-64x}{(8x^2 + 2) \cdot \sqrt{(8x^2 + 2)}} = \frac{-32x}{(4x^2 + 1) \cdot \sqrt{(8x^2 + 2)}}$
1.11)	$[\sqrt{\sin(x)}]' = \left[\sin^{\frac{1}{2}}(x)\right]' = \frac{1}{2} \cdot \sin^{-\frac{1}{2}}(x) \cdot \cos(x) = \frac{\cos(x)}{2 \cdot \sqrt{\sin(x)}}$
1.12)	$[\sqrt{\sin(2x)}]' = \left[\sin^{\frac{1}{2}}(2x)\right]' = \frac{1}{2} \cdot \sin^{-\frac{1}{2}}(2x) \cdot \cos(2x) \cdot 2 = \frac{\cos(2x)}{\sqrt{\sin(2x)}}$
1.13)	$\left[\cos\left(\frac{x}{1+x}\right)\right]' = \left[-\sin\left(\frac{x}{1+x}\right)\right]' \cdot \frac{1 \cdot (1+x) - x \cdot 1}{(1+x)^2} = \frac{-\sin\left(\frac{x}{1+x}\right)}{(1+x)^2}$
1.14)	$[\tan(\sqrt{x+1})]' = \frac{1}{\cos^2(\sqrt{x+1})} \cdot (\sqrt{x+1})' = \frac{1}{\cos^2(\sqrt{x+1})} \cdot \frac{1}{2 \cdot \sqrt{x+1}} \cdot (x+1)' = \frac{1}{2 \cdot \sqrt{x+1} \cdot \cos^2(\sqrt{x+1})}$

2 Calculez les dérivées des fonctions suivantes :

2.1) $\left[e^{3x+2} \right]' = e^{3x+2} \cdot (3x+2)' = 3 \cdot e^{3x+2}$	2.2) $\left[e^{x^2} \right]' = e^{x^2} \cdot (x^2)' = 2x \cdot e^{x^2}$
2.3) $\left[e^{\cos(x)} \right]' = e^{\cos(x)} \cdot (\cos(x))' = -\sin(x) \cdot e^{\cos(x)}$	2.4) $\left[x \cdot e^x \right]' = 1 \cdot e^x + x \cdot e^x = (x+1) \cdot e^x$

2.5) $\left[x \cdot e^{x^2+3x} \right]' = 1 \cdot e^{x^2+3x} + x \cdot e^{x^2+3x} \cdot (2x+3) = (2x^2 + 3x + 1) \cdot e^{x^2+3x}$
2.6) $\left[\cos(x) \cdot e^x \right]' = -\sin(x) \cdot e^x + \cos(x) \cdot e^x = (-\sin(x) + \cos(x)) \cdot e^x$
2.7) $\left[\sin(x) \cdot e^{x^2} \right]' = \cos(x) \cdot e^{x^2} + \sin(x) \cdot e^{x^2} \cdot 2x = (\cos(x) + 2x \cdot \sin(x)) \cdot e^{x^2}$
2.8) $\left[\sin^2(x) \cdot e^{3x} \right]' = 2 \cdot \sin(x) \cdot \cos(x) \cdot e^{3x} + \sin^2(x) \cdot e^{3x} \cdot 3 = (2 \cdot \cos(x) + 3 \sin(x)) \cdot \sin(x) \cdot e^{3x}$

3 Calculez les dérivées des fonctions suivantes :

3.1) $\left[\ln(3x+2) \right]' = \frac{1}{3x+2} \cdot (3x+2)' = \frac{3}{3x+2}$	3.2) $\left[\ln(x^2) \right]' = \frac{1}{x^2} \cdot (x^2)' = \frac{2x}{x^2} = \frac{2}{x}$ Quelle propriété des logarithmes est en jeu ?
3.3) $\left[\ln(1-x^3) \right]' = \frac{1}{1-x^3} \cdot (1-x^3)' = \frac{-3x^2}{1-x^3}$	3.4) $\left[x \cdot \ln(x) \right]' = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$
3.5) $\left[x^2 \cdot \ln(x) \right]' = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} = x \cdot (2 \ln(x) + 1)$	3.6) $\left[x \cdot \ln(x^2) \right]' = 1 \cdot \ln(x^2) + x \cdot \frac{1}{x^2} \cdot 2x = \ln(x^2) + 2$
3.7) $\left[\ln(1+e^x) \right]' = \frac{1}{1+e^x} \cdot e^x = \frac{e^x}{1+e^x}$	3.8) $\left[\ln\left(\frac{1}{x}\right) \right]' = \frac{1}{\frac{1}{x}} \cdot \frac{-1}{x^2} = x \cdot \frac{-1}{x^2} = \frac{-1}{x}$ Quelle propriété des logarithmes est en jeu ?

3.9) $\left[\ln(\cos^2(x)+1) \right]' = \frac{1}{\cos^2(x)+1} \cdot 2 \cdot \cos(x) \cdot (-\sin(x)) = \frac{-2 \cdot \sin(x) \cdot \cos(x)}{\cos^2(x)+1}$
3.10) $\left[\ln\left(\frac{1}{1+x^2}\right) \right]' = \frac{1}{\frac{1}{1+x^2}} \cdot \left[(1+x^2)^{-1} \right]' = \frac{1+x^2}{1} \cdot (-1) \cdot (1+x^2)^{-2} \cdot 2x = \frac{-2x \cdot (1+x^2)}{(1+x^2)^2} = \frac{-2x}{1+x^2}$ Autre manière : $\left[\ln\left(\frac{1}{1+x^2}\right) \right]' = \left[-\ln(1+x^2) \right]' = -\frac{1}{1+x^2} \cdot (1+x^2)' = \frac{-2x}{1+x^2}$