❶ Calculez les dérivées des fonctions suivantes :

1.1	$\left[\sin^3(x)\right]' = 3\cdot\sin^2(x)\cdot\cos(x)$
1.2	$\left[-2\cdot\sin^4(x)\right]' = -2\cdot 4\cdot\sin^3(x)\cdot\cos(x) = -8\cdot\sin^3(x)\cdot\cos(x)$
1.3	$\left[x\cdot\cos(x)\right]' = 1\cdot\cos(x) + x\cdot(-\sin(x)) = \cos(x) - x\cdot\sin(x)$
1.4	$\left[-\frac{3x\cdot\cos(x)}{7}\right]' = -\frac{3}{7}\cdot\left[x\cdot\cos(x)\right]' = -\frac{3}{7}\cdot\left[\cos(x) - x\cdot\sin(x)\right]$
1.5	$\left[5\cdot\cos^2(x) - \frac{3}{x}\right]' = 5\cdot 2\cdot\cos(x)\cdot\left(-\sin(x)\right) - 3\cdot\frac{-1}{x^2} = -10\cdot\sin(x)\cdot\cos(x) + \frac{3}{x^2}$
1.6	$\left[\left(1+x^3\right)\cdot\sin(x)\right]' = 3x^2\cdot\sin(x) + \left(1+x^3\right)\cdot\cos(x)$
1.7	$\left[\sin^2(x) + \cos^2(x)\right]' = \left[1\right]' = 0$ Avec les règles habituelles : même réponse !
1.8	$\left[\sin^2(x) - \cos^2(x)\right]' = 2\cdot\sin(x)\cdot\cos(x) - 2\cdot\cos(x)\cdot\left(-\sin(x)\right) = \underline{\underline{4\cdot\sin(x)\cdot\cos(x)}} \overset{\text{identité trigonométrique}}{=} \underline{\underline{2\cdot\sin(2x)}}$
1.9	$\left[2\cdot\sin(x)\cdot\cos(x)\right]' = 2\cdot\left[\sin(x)\cdot\cos(x)\right]' = 2\cdot\left[\cos(x)\cdot\cos(x) + \sin(x)\cdot\left(-\sin(x)\right)\right] = 2\cdot\left[\cos^2(x) - \sin^2(x)\right]$ $\left[2\cdot\sin(x)\cdot\cos(x)\right]' \overset{\text{trigo}}{=} \left[\sin(2x)\right]' = \cos(2x)\cdot 2 = \underline{\underline{2\cdot\cos(2x)}} \overset{\text{trigo}}{=} \underline{\underline{2\cdot\left[\cos^2(x) - \sin^2(x)\right]}}$
1.10	$\left[\frac{\cos(x)\cdot\sin^2(x)}{4}\right]' = \left[\frac{1}{4}\cdot\cos(x)\cdot\sin^2(x)\right]' = \frac{1}{4}\cdot\left[\cos(x)\cdot\sin^2(x)\right]' =$ $\frac{1}{4}\cdot\left[\left(-\sin(x)\right)\cdot\sin^2(x) + \cos(x)\cdot 2\cdot\sin(x)\cdot\cos(x)\right] = \frac{1}{4}\cdot\left[-\sin^3(x) + 2\cdot\sin(x)\cdot\cos^2(x)\right]$

❷ Calculez les dérivées des fonctions suivantes :

2.1	$\left[\frac{x+1}{x+2}\right]' = \frac{(x+1)'\cdot(x+2) - (x+1)\cdot(x+2)'}{(x+2)^2} = \frac{1\cdot(x+2) - (x+1)\cdot 1}{(x+2)^2} = \frac{1}{(x+2)^2}$
2.2	$\left[\frac{x+3}{x-4}\right]' = \frac{1\cdot(x-4) - (x+3)\cdot 1}{(x-4)^2} = \frac{-7}{(x-4)^2}$
2.3	$\left[\frac{x+7}{2-x}\right]' = \frac{1\cdot(2-x) - (x+7)\cdot(-1)}{(2-x)^2} = \frac{2-x+x+7}{(2-x)^2} = \frac{9}{(2-x)^2}$
2.4	$\left[\frac{5x-3}{1-2x}\right]' = \frac{5\cdot(1-2x) - (5x-3)\cdot(-2)}{(1-2x)^2} = \frac{5-10x+10x-6}{(1-2x)^2} = \frac{-1}{(1-2x)^2}$
2.5	$\left[\frac{x^2-1}{x^2+1}\right]' = \frac{2x\cdot(x^2+1) - (x^2-1)\cdot 2x}{(x^2+1)^2} = \frac{2x\cdot(x^2+1-x^2+1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$
2.6	$\left[\frac{4x^2-9}{x^2+3}\right]' = \frac{8x\cdot(x^2+3) - (4x^2-9)\cdot 2x}{(x^2+3)^2} = \frac{2x\cdot(4x^2+12-4x^2+9)}{(x^2+3)^2} = \frac{42x}{(x^2+3)^2}$

2.7	$\left[\dfrac{x}{1+x^2}\right]' = \dfrac{1\cdot(1+x^2)-x\cdot 2x}{(1+x^2)^2} = \dfrac{1+x^2-2x^2}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$
2.8	$\left[\dfrac{x^3+2}{1-7x}\right]' = \dfrac{3x^2\cdot(1-7x)-(x^3+2)\cdot(-7)}{(1-7x)^2} = \dfrac{3x^2-21x^3+7x^3+14}{(1-7x)^2} = \dfrac{-14x^3+3x^2+14}{(1-7x)^2}$
2.9	$\left[\dfrac{2}{(x-1)^3}\right]' = 2\cdot\left[(x-1)^{-3}\right]' = 2\cdot(-3)\cdot(x-1)^{-4}\cdot(x-1)' = \dfrac{-6}{(x-1)^4}$
2.10	$\left[\dfrac{(1-x)^2}{x^3}\right]' = \dfrac{2\cdot(1-x)\cdot(-1)\cdot x^3-(1-x)^2\cdot 3x^2}{x^6} = \dfrac{(1-x)\cdot\left[-2x^3-(1-x)\cdot 3x^2\right]}{x^6} = \dfrac{(1-x)\cdot(-2x^3-3x^2+3x^3)}{x^6}$ $= \dfrac{(1-x)\cdot(x^3-3x^2)}{x^6} = \dfrac{(1-x)\cdot(x-3)\cdot x^2}{x^6} = \underline{\underline{\dfrac{(1-x)\cdot(x-3)}{x^4}}}$ $\left(= \dfrac{-x^2+4x-3}{x^4}\right)$ *forme factorisée préférable* Autre méthode : $\left[\dfrac{(1-x)^2}{x^3}\right]' = \left[\dfrac{1-2x+x^2}{x^3}\right]' = \left[x^{-3}-2x^{-2}+x^{-1}\right]' = -3x^{-4}+4x^{-3}-x^{-2} = \dfrac{-3+4x-x^2}{x^4}$
2.11	$\left[\dfrac{x^2-6}{x^2+x+2}\right]' = \dfrac{2x\cdot(x^2+x+2)-(x^2-6)\cdot(2x+1)}{(x^2+x+2)^2} = \dfrac{2x^3+2x^2+4x-2x^3+12x-x^2+6}{(x^2+x+2)^2} = \dfrac{x^2+16x+6}{(x^2+x+2)^2}$
2.12	$\left[\dfrac{x^2+x+3}{x^2+3x+1}\right]' = \dfrac{(2x+1)\cdot(x^2+3x+1)-(x^2+x+3)\cdot(2x+3)}{(x^2+3x+1)^2} =$ $\dfrac{2x^3+6x^2+2x+x^2+3x+1-2x^3-2x^2-6x-3x^2-3x-9}{(x^2+3x+1)^2} = \dfrac{2x^2-4x-8}{(x^2+3x+1)^2}$

❸ Calculez les dérivées des fonctions suivantes :

3.1	$\left[\dfrac{1}{\sin(x)}\right]' = \left[\sin^{-1}(x)\right]' = -\sin^{-2}(x)\cdot\cos(x) = \dfrac{-\cos(x)}{\sin^2(x)}$
3.2	$\left[\dfrac{1}{\cos^2(x)}\right]' = \left[\cos^{-2}(x)\right]' = -2\cdot\cos^{-3}(x)\cdot(-\sin(x)) = \dfrac{2\cdot\sin(x)}{\cos^3(x)}$
3.3	$\left[\dfrac{\sin(x)}{\cos(x)}\right]' = \dfrac{\cos(x)\cdot\cos(x)-\sin(x)\cdot(-\sin(x))}{\cos^2(x)} = \dfrac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \dfrac{\cos^2(x)}{\cos^2(x)}+\dfrac{\sin^2(x)}{\cos^2(x)} = 1+\tan^2(x)$ ou $= \dfrac{1}{\cos^2(x)}$
3.4	$\left[\dfrac{1}{\sin^3(x)}\right]' = \left[\sin^{-3}(x)\right]' = -3\cdot\sin^{-4}(x)\cdot\cos(x) = \dfrac{-3\cdot\cos(x)}{\sin^4(x)}$
3.5	$\left[\dfrac{\sin^2(x)}{\cos^2(x)}\right]' = \left[\tan^2(x)\right]' = 2\cdot\tan(x)\cdot(1+\tan^2(x))$ ou $= 2\cdot\tan(x)\cdot\dfrac{1}{\cos^2(x)} = \dfrac{2\cdot\sin(x)}{\cos^3(x)}$ cf. ***3.3***

3.6	$\left[\dfrac{2}{\tan(x)}\right]' = \left[2\cdot\tan^{-1}(x)\right]' = -2\cdot\tan^{-2}(x)\cdot(1+\tan^2(x)) = \dfrac{-2\cdot(1+\tan^2(x))}{\tan^2(x)}$ ou $= -2\cdot\tan^{-2}(x)\cdot\dfrac{1}{\cos^2(x)} = \dfrac{-2\cdot\cos^2(x)}{\sin^2(x)}\cdot\dfrac{1}{\cos^2(x)} = \dfrac{-2}{\sin^2(x)}$
3.7	$\left[\dfrac{-8}{\cos^2(x)}\right]' = -8\cdot\left[\cos^{-2}(x)\right]' = -8\cdot(-2)\cdot\cos^{-3}(x)\cdot\left(-\sin(x)\right) = \dfrac{-16\cdot\sin(x)}{\cos^3(x)}$
3.8	$\left[\dfrac{15}{\sin^3(x)}\right]' = -45\cdot\sin^{-4}(x)\cdot\cos(x) = \dfrac{-45\cdot\cos(x)}{\sin^4(x)}$ cf. ***3.4***
3.9	$\left[\dfrac{\cos^2(x)}{1+x^2}\right]' = \dfrac{2\cdot\cos(x)\cdot\left(-\sin(x)\right)\cdot(1+x^2)-\cos^2(x)\cdot 2x}{\left(1+x^2\right)^2} = \dfrac{-2\cdot\cos(x)\cdot\left[(1+x^2)\cdot\sin(x)+x\cdot\cos(x)\right]}{\left(1+x^2\right)^2}$
3.10	$\left[\dfrac{-3}{\tan^2(x)}\right]' = -3\cdot\left[\tan^{-2}(x)\right]' = -3\cdot(-2).\tan^{-3}(x)\cdot\left[1+\tan^2(x)\right] = \dfrac{6\cdot(1+\tan^2(x))}{\tan^3(x)}$ ou $= 6\cdot\dfrac{\cos^3(x)}{\sin^3(x)}\cdot\dfrac{1}{\cos^2(x)} = \dfrac{6\cdot\cos(x)}{\sin^3(x)}$