❶ 1) $\log(1'000) = \log(10^3) = 3$ 2) $\log(100'000) = \log(10^5) = 5$

3) $\log(0{,}0001) = \log(10^{-4}) = -4$ 4) $\log\left(\sqrt{10}\right) = \log\left(10^{1/2}\right) = 1/2$

5) $\log\left(\sqrt[3]{10}\right) = \log\left(10^{1/3}\right) = 1/3$ 6) $\log\left(\sqrt[3]{100}\right) = \log\left(10^{2/3}\right) = 2/3$

7) $Log_2(32) = Log_2(2^5) = 5$ 8) $Log_3(81) = Log_3(3^4) = 4$

9) $Log_5(625) = Log_5(5^4) = 4$ 10) $Log_2(1/64) = Log_2(2^{-6}) = -6$

11) $Log_2\left(\sqrt{8}\right) = Log_2\left(2^{3/2}\right) = 3/2$ 12) $\log\left(100 \cdot \sqrt{10}\right) = \log\left(10^2 \cdot 10^{1/2}\right) = \log\left(10^{2,5}\right) = 2{,}5$

13) $\log\left(\sqrt[5]{100} \cdot \sqrt[7]{1'000}\right) = \log\left(10^{2/5} \cdot 10^{3/7}\right) = \log\left(10^{2/5+3/7}\right) = \frac{2}{5} + \frac{3}{7} = \frac{14+15}{35} = \frac{29}{35}$ Autre possibilité :

13) $\log\left(\sqrt[5]{100} \cdot \sqrt[7]{1'000}\right) = \log\left(10^{2/5} \cdot 10^{3/7}\right) = \log\left(10^{2/5}\right) + \log\left(10^{3/7}\right) = \frac{2}{5} + \frac{3}{7} = \frac{14+15}{35} = \frac{29}{35}$

14) $\ln(e^{-4,5}) = -4{,}5$ 15) $\ln(x) = 0 \Rightarrow x = 1$

16) $\ln(x) = 1 \Rightarrow x = e^1 = e \approx 2{,}718281828459$ 17) $\ln(x) = -2 \Rightarrow x = e^{-2} \approx 0{,}1353352832366$

18) $Log_x(25) = 2 \Rightarrow 0{,}5 \cdot Log_x(25) = 1 \Rightarrow Log_x(25^{0,5}) = 1 \Rightarrow Log_x(5) = 1 \Rightarrow x = 5$

19) $Log_x(256) = 4 \Rightarrow 0{,}25 \cdot Log_x(256) = 1 \Rightarrow Log_x(256^{0,25}) = 1 \Rightarrow Log_x(4) = 1 \Rightarrow x = 4$

20) $Log_x(\sqrt{10}) = 1/2 \Rightarrow 2 \cdot Log_x(\sqrt{10}) = 1 \Rightarrow Log_x(\sqrt{10}^2) = 1 \Rightarrow Log_x(10) = 1 \Rightarrow x = 10$

21) $x = e^{\ln(x)} = 0{,}5$ 22) $e^{5 \cdot \ln(2)} = e^{\ln(2^5)} = 2^5 = 32$

23) $10^{\log(7)} = 7$ 24) $10^{\log(16) - 1} = 10^{\log(16)} \cdot 10^{-1} = 16 \cdot 0{,}1 = 1{,}6$

25) $x = Log_b\left(b^3\right) = 3 \cdot Log_b(b) = 3 \cdot 1 = 3$

26) $x = \ln\left(\sqrt{e}\right) = \ln\left(e^{1/2}\right) = (1/2) \cdot \ln(e) = 1/2 = 0{,}5$

27) $\log(x-1) = 1 \Leftrightarrow x - 1 = 10^1 \Leftrightarrow x = 11$

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❷ a) $\ln(10) \approx 2{,}302585$ b) $\log(e) \approx 0{,}4342945$ c) $10 \cdot \log(2) \approx 3{,}010300$

d) $\ln(100) \approx 4{,}60517$ e) $\log(e^2) \approx 0{,}868590$ f) $10 \cdot \log(4) \approx 6{,}020600$

d) $\ln(100) = \ln(10^2) = 2 \cdot \ln(10)$ e) $\log(e^2) = 2 \cdot \log(e)$ f) $10 \cdot \log(4) = 2 \cdot 10 \cdot \log(2)$

Les résultats de la deuxième ligne sont toujours le double de ceux de la première ligne.

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❸ Puisque $2^{10} \approx 10^3$, on a : $10 \cdot \log(2) = \log(2^{10}) \approx \log(10^3) = 3$

C'est une approximation à 0,35% près du résultat obtenu en ❷ c).

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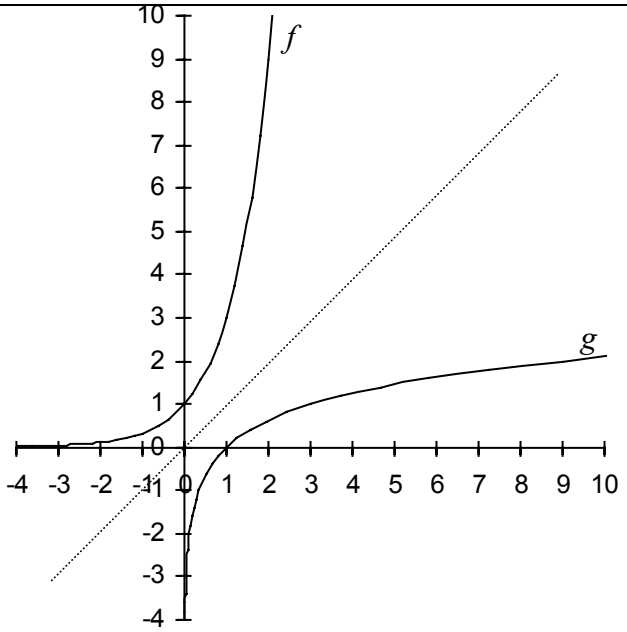
❹



$f(x) = 3^x$ $g(x) = \text{Log}_3(x)$

La fonction g est la réciproque de la fonction f.